

The University of the State of New York

284TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, June 18, 1942 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II and III

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A , B and C represent the angles of a triangle ABC ; a , b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two parts, including at least *two* questions from each part.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 Express 210° in radians. [Answer may be left in terms of π .] 1.....
- 2 Does -30° satisfy the equation $2 \sin x + 1 = 0$? [Answer *yes* or *no*.] 2.....
- 3 Find the smallest positive value of x for which $\sin x = -\frac{1}{2}$ and $\cos x = -\frac{\sqrt{3}}{2}$ 3.....
- 4 Find the positive value of $\tan (\sin^{-1} \frac{1}{7})$ 4.....
- 5 Find, correct to the *nearest tenth*, the number whose logarithm is 2.1347 5.....
- 6 If $\log \tan x = 9.8355 - 10$ and x is a positive acute angle, find the value of x correct to the *nearest minute*. 6.....
- 7 Find the value of $\cos 42^\circ 22'$ 7.....
- 8 Express $\cos 60^\circ + \cos 40^\circ$ in terms of $\cos 50^\circ$ and $\cos 10^\circ$. 8.....
- 9 Which of the six functions of an angle have positive values when the angle is in the third quadrant? 9.....
- 10 Express $\tan (45^\circ - y)$ in terms of $\tan y$. 10.....
- 11 If $\cos A = -\frac{1}{3}$ and A is obtuse, find the value of $\sin 2A$. 11.....
- 12 If $\tan x = \frac{3}{4}$ and x is acute, find the value of $\sin \frac{1}{2}x$. [Answer may be left in radical form.] 12.....
- 13 Is the following statement *true* or *false*: There are two different triangles in which $a = 10$, $c = 8$ and $A = 50^\circ$? 13.....
- 14 While flying at a height of 1000 feet, an aviator observed that the angle of depression of an enemy airport was 26° . How far, correct to the *nearest foot*, is the airport from the point on the ground directly below the aviator? 14.....
- 15 The base of an isosceles triangle is 20 inches and one of the base angles is 54° . Find the altitude of the triangle correct to the *nearest inch*. 15.....

Directions (questions 16-20) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a , b or c .

- 16 If A is acute and $\sin A = \frac{3}{8}$, then the value of $\cos A$ is (a) $\frac{3}{8}$, (b) less than $\frac{3}{8}$ or (c) greater than $\frac{3}{8}$. 16.....

17 If x is limited to positive values less than 360° , the statement $\cos x = \sin 30^\circ$ is true for (a) only one value of x , (b) two values of x or (c) more than two values of x .

17.....

18 As x varies from 0° to 90° , the value of $\cos 2x$ (a) decreases, (b) decreases and then increases or (c) increases.

18.....

19 If A and B are complementary angles, then (a) $\sin A \times \csc B = 1$, (b) $\sin A = \cos B$ or (c) $\sin A + \sin B = 1$

19.....

20 The minimum value of $\sin 3x$ is (a) 0, (b) -1 or (c) -3 .

20.....

See instructions for parts II and III on page 1.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from this part.

- 21 a Prove the identity: $\cos 2A = \frac{\cot^2 A - 1}{\cot^2 A + 1}$ [5]
 b Find, correct to the *nearest degree*, the positive acute angle which satisfies the equation $6 \cos^2 x + \cos x - 1 = 0$ [5]
- 22 a Starting with the formula for $\tan(x + y)$, derive the formula for $\tan 2x$ in terms of $\tan x$. [4]
 b Prove geometrically that $\cos(180^\circ + A) = -\cos A$ for the case in which A is acute. [6]
- 23 In triangle ABC , $a \sin A = b \sin B$. Prove that triangle ABC is isosceles. [10]
- 24 a Draw the graph of $y = \sin x$ as x varies from 0° to 180° inclusive at intervals of 30° . [3]
 b Using the same axes as in a, draw the graph of $y = \sin 2x$ as x varies from 0° to 180° inclusive at intervals of 15° . [5]
 c In the equation $y = \sin nx$, n takes the values 1, 2, 3, etc. If for each of these values of n the equation $y = \sin nx$ were plotted from $x = 0^\circ$ to $x = 180^\circ$, would the number of intersections of these graphs with the x axis remain constant, increase or decrease, as n increases? [2]
- *25 a Write the name of the graph whose equation in polar coordinates is
 (1) $r = 5$ [2]
 (2) $\tan \theta = 1$ [2]
 b Show that the equation $xy = 4$, when transformed into polar coordinates, is $r^2 \sin 2\theta = 8$ [6]

Part III

Answer at least two questions from this part.

- 26 In triangle ABC , $a = 46$, $b = 62$, $c = 71$. Find B correct to the *nearest minute*. [10]
- 27 The decorative design on an auditorium floor has the form of a regular octagon circumscribed about a circle. Each side of the octagon is 12.0 inches.
 a Find the radius of the circle correct to the *nearest tenth* of an inch. [6]
 b Using the radius found in answer to a, find the area of the octagon. [4]
- 28 Two forces, one of 70 pounds and the other of 40 pounds, act on a body at an angle of $68^\circ 40'$. Find, correct to the *nearest minute*, the angle between the resultant and the greater force. [10]
- 29 Points A and B are on opposite banks of a river whose width is represented by line segment AB . Point C is on the same side of the river as B , in line with AB and 140 feet from B . A tree standing at A subtends an angle of $35^\circ 20'$ at B and an angle of $19^\circ 10'$ at C . Find the width of the river correct to the *nearest foot*. [10]

* This question is based on one of the optional topics in the syllabus.