# THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS 

 COURSE IThursday, January 28, 1993 - 9:15 a.m. to 12:15 p.m., only

## Notice . . .

Calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

## Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of $\pi$ or in radical form. [60]

1 A bag contains three red marbles, two green marbles, and four orange marbles. If one marble is selected at random, what is the probability that it is not orange?

2 In the accompanying diagram, $\triangle A B C \sim \triangle D E F$, $A B=12, A C=8, D E=x$, and $D F=2$. Find the value of $x$.


3 Let $p$ represent "I go to the movies" and let $q$ represent "I study my math." Using $p$ and $q$, write in symbolic form: "If I do not study my math, then I do not go to the movies."

4 Express the sum of $3 x^{2}-2 x+5$ and $x^{2}+2 x-8$ in simplest form.

5 Solve for $x$ : $7 x-2=5 x+3$
6 Express $4 x^{2}-25$ as the product of two binomials.

7 In the accompanying diagram, $\overrightarrow{D H} \| \overrightarrow{E F}$. If $\mathrm{m} \angle H D F=30$ and $\mathrm{m} \angle E D F=55$, find $\mathrm{m} \angle E$.


8 Without repetition of digits, how many different three-digit numbers less than 800 can be formed using the numbers in the set $\{2,3,9\}$ ?

9 A census taker visited 100 homes. The accompanying table shows the frequencies for the number of people living in each home. Which interal contains the median for these data?
Number of People
Living in One Home

| Interal | Frequency |
| :---: | :---: |
| $1-2$ | 40 |
| $3-4$ | 39 |
| $5-6$ | 16 |
| --8 | 5 |

10 Find in terms of $x$, the mean of $3 x-5.5 x-6$, and $4:-11$.

11 What is the product of $\frac{1}{2} x^{3} y$ and $\frac{1}{4} x y^{2}$ ?
12. Solve for $m: \frac{m+3}{2 m}=\frac{4}{5}, m \neq 0$

13 In the accompanying diagram, the length of the diameter of circle $O$ equals the length of a side of the square. If the circumference of the circle is $6 \pi$. what is the perimeter of the square?


14 Perform the indicated operations and express as a binomial:

$$
-3(x+2)-x
$$

15 Express the product of $2 a+1$ and $a+5$ as a trinomial.

16 In the accompanying figure, $\triangle A B C$ has coordinates $A(0,3), B(7,3)$, and $C(7,7)$. Find the area of $\triangle A B C$.


17 In the accompanying diagram of rectangle $A B C D, A D=12$ and $A B=5$. What is the length of diagonal $\overline{A C}$ ?


18 Add: $\frac{3}{2 x}+\frac{2}{5 x}, x \neq 0$

19 Solve for $x$ : $\quad 0.3 x-2=0.7$

Directions (20-35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

20 When $12 x^{4}-3 x^{3}+6 x^{2}$ is divided by $3 x^{2}$, the quotient is
(1) $4 x^{2}-3 x^{3}+6 x^{2}$
(3) $9 x^{2}-x+2$
(2) $12 x^{4}-3 x^{3}+2$
(4) $4 x^{2}-x+2$

21 If a regular pentagon has a side of length $2 x+1$, what is the perimeter of the pentagon?
(1) $4 x^{2}+1$
(3) $10 x+1$
(2) $10 x+5$
(4) $10 x^{2}+5$

22 Which type of symmetry does a square have?
(1) line symmetry, only
(2) point symmetry, only
(3) both line and point symmetry
(4) no symmetry

23 The interest on a loan varies directly as the rate. If the rate is halved, then the interest
(1) is halved
(3) remains the same
(2) is doubled
(4) is multiplied by 4

24 Which is an equation for line $\ell$ in the accompanying diagram?

(1) $y=2 x+2$
(3) $y=-2 x-4$
(2) $y=2 x-4$
(4) $y=-2 x+2$

25 One member of the solution set of $3 x-1 \geq 4$ is
(1) 1
(3) $\frac{5}{3}$
(2) $\frac{2}{3}$
(4) $-\frac{4}{3}$

26 If two angles of a triangle are complementary, then the triangle must be
(1) obtuse
(3) isosceles
(2) equilateral
(4) right

27 In the accompanying diagram, $\triangle A B C$ is a right triangle.


Which diagram below represents the image of $\triangle A B C$ when rotated $90^{\circ}$ counterclockwise about the origin?
(1)

(3)

(2)

(4)


28 Which figure has the largest area?
(1) a square whose side measures 6
(2) a circle whose diameter measures 6
(3) a triangle whose base and height each measure 6
(4) an equilateral triangle whose side measures 6

29 Written in factored form, the trinomial $3 x^{2}+5 x-2$ is equivalent to
(1) $(3 x+1)(x-2)$
(3) $(3 x+2)(x-1)$
(2) $(3 x-1)(x+2)$
(4) $(3 x-2)(x+1)$

30 Which statement would be a correct heading for the last column of the table?

| $p$ | $q$ | $\sim q$ | $\vdots$ |
| :---: | :---: | :---: | :---: |
| T | T | F | F |
| T | F | T | T |
| F | T | F | T |
| F | F | T | F |

(1) $q \rightarrow \sim q$
(3) $p-\sim q$
(2) $p \wedge \sim q$
(4) $p \vee \sim q$

31 The expression $5 \sqrt{8}-3 \sqrt{2}$ is equivalent to
(1) 7
(3) $2 \sqrt{6}$
(2) $7 \sqrt{2}$
(4) $\sqrt{34}$

32 If 0.0000055 is expressed in the form $5.5 \times 10^{n}$, what is the value of $n$ ?
(1) 5
(3) -5
(2) 6
(4) -6

33 The probability of throwing two fours on a single toss of a pair of dice is
(1) $\frac{1}{36}$
(3) $\frac{1}{6}$
(2) $\frac{1}{12}$
(4) $\frac{1}{3}$

34 If $x=-2$ and $y>0$, in which quadrant of the coordinate plane is point $P(x, y)$ located?
(1) I
(3) III
(2) II
(4) IV

35 The converse of $\sim p \rightarrow q$ is
(1) $p \rightarrow \sim q$
(3) $q \rightarrow \sim p$
(2) $\sim q \rightarrow p$
(4) $\sim p \rightarrow \sim q$

Answers to the following questions are to be written on paper provided by the school.

## Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]
$36 a$ On the same set of coordinate axes, graph the following system of inequalities and label the solution set $S$.

$$
\begin{align*}
y & \geq 3 x+1  \tag{8}\\
x+2 y & <5
\end{align*}
$$

$b$ Write the coordinates of a point in the solution set of the system of inequalities graphed in part $a$.
[2]
37 A fair die and a fair coin are tossed.
a Draw a tree diagram or list the sample space for all possible outcomes. [2]
$b$ Find the probability of getting
(1) an odd number and a head [2]
(2) a number greater than 5 and a tail
(3) a number greater than 4 or a head
(4) a prime number and either a head or a tail [2]

38 The denominator of a fraction is four less than twice the numerator. If three is added to both the numerator and the denominator, the new fraction is equal to $\frac{2}{3}$. Find the original fraction. [Only an algebraic solution will be accepted.] [5.5]

39 A taxi ride costs $\$ 1.25$ for the first quarter of a mile and $\$ 0.30$ for each subsequent quarter of a mile.
a A ride that costs $\$ 5.45$ is how many miles longer than a ride that costs $\$ 3.95$ ? [Show or explain the procedure used to obtain your answer.] [6]
$b$ If $n$ represents the number of quarter miles traveled and $C$ represents the total cost of the trip, which formula could be used to find $C$ ?
[2]
(1) $C=1.25 n$
(2) $C=1.25+0.30 n$
(3) $C=1.25+0.30(n-1)$
(4) $C=1.25 n+0.30 n$
$c$ Using your answer from part $b$, find the value of $C$ if $n=20$. [2]

40 Solve the following system of equations algebraically and check:

$$
\begin{align*}
& 3 x+5 y=4  \tag{8,2}\\
& 4 x+3 y=-2
\end{align*}
$$

41 A small company recorded the number of hours each of their 21 part-time employees worked during one week. The hours employees worked were: $14,22,13,2,7,13,18,29,19,15,9,16$, $23,12,17,28,20,4,18,8,24$.
a On your answer paper, copy and complete the tables below using the given data.

| Interval <br> (in hours) | Frequency |
| :---: | :---: |
| $26-30$ |  |
| $21-25$ |  |
| $16-20$ |  |
| $11-15$ |  |
| $6-10$ |  |
| $0-5$ | 2 |


| Interval <br> (in hours) | Cumulative <br> Frequency |
| :---: | :---: |
| $0-30$ |  |
| $0-25$ |  |
| $0-20$ |  |
| $0-15$ |  |
| $0-10$ |  |
| $0-5$ | 2 |

$b$ Which interval in the frequency table contains the median number of hours worked? [2]
$c$ Construct a cumulative frequency histogram using the data from the appropriate table.
[4]

42 a On your answer paper, copy and complete the truth table for the statement $\sim p \rightarrow q-(p \wedge \sim q)$. [8]

| $p$ | $q$ | $p \rightarrow q$ | $\sim(p \rightarrow q)$ | $\sim q$ | $p \wedge \sim q$ | $\sim p-q-p \cdot \sim q$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T | T |  |  |  |  |  |
| T | F |  |  |  |  |  |
| F | T |  |  |  |  |  |
| F | F |  |  |  |  |  |

$b$ Is the statement $\sim(p \rightarrow q) \leftrightarrow(p \wedge \sim q)$ a tautology?
[1]
$c$ Justify the answer given in part $b$.
[1]

