The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Monday, June 18, 1984 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

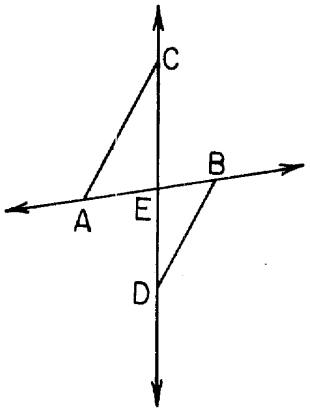
Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form.

1 If $x * y$ is defined as $x^2 - 3y$, find the value of $4 * 2$.

2 At a certain point in a game, Nancy needs to roll either a 2 or a 3 on a die to win. What is the probability of her rolling either a 2 or a 3 on one roll of a fair six-sided die?

3 The lengths of corresponding sides of two similar polygons are in the ratio 2:5. If the perimeter of the larger polygon is 100, what is the perimeter of the smaller polygon?

4 As shown in the accompanying diagram, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point E, and $\overline{CA}$ and $\overline{BD}$ are drawn. If $\overline{CA} \parallel \overline{BD}$, $m\angle DEB = 100$, and $m\angle BDE = 30$, find $m\angle CAE$.

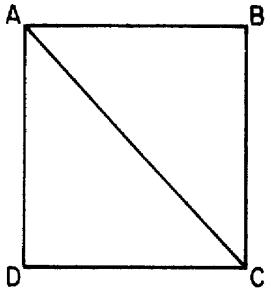


5 What is the positive root of the equation $x^2 - 5x = 14$?

6 The measures of the angles of a triangle are in the ratio of 1:3:5. Find the number of degrees in the measure of the *smallest* angle of the triangle.

7 In $\triangle ABC$, D is a point on $\overline{CA}$, E is a point on $\overline{CB}$, and $\overline{DE} \parallel \overline{AB}$. If $CD = 4$, $CA = 6$, and $BC = 9$, find CE.

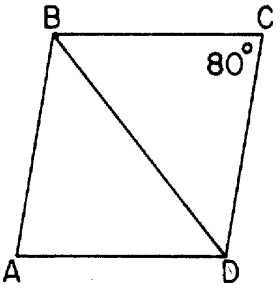
8 In the accompanying diagram, the length of a side of square $ABCD$ is 3. Find, in radical form, the length of $\overline{AC}$.



9 In isosceles triangle ABC, base $\overline{AC}$ is extended through C to D and $m\angle BCD = 110$. What is the measure of vertex angle B?

10 The measure of the length of a rectangle is three times the measure of the width, and the perimeter is 32. Find the area of the rectangle.

11 In the accompanying diagram of rhombus $ABCD$, $m\angle BCD = 80$. Find $m\angle BDA$.



12 How many different arrangements of 5 letters can be made from the letters in the word "ELVES"?

13 The endpoints of $\overline{AB}$ are $A(x,3)$ and $B(4,7)$. If the coordinates of the midpoint M of $\overline{AB}$ are $(-1,5)$, find x.

14 In right triangle ABC, altitude $\overline{CD}$ is drawn to the hypotenuse. If $AD = 4$ and $DB = 5$, find AC.

Directions (15–34): For each question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 What is the value of $b \# (a \# e)$ within the system below?

#	a	b	c	d	e
a	a	e	c	e	a
b	e	a	c	a	e
c	c	c	c	c	c
d	e	a	c	a	e
e	a	e	c	e	a

(1) e
(2) b
(3) c
(4) d

16 The negation of $r \wedge \sim t$ is
(1) $\sim r \vee t$
(2) $\sim r \wedge t$
(3) $\sim r \vee \sim t$
(4) $\sim r \wedge \sim t$

17 If the equation $x^2 - 3x + c = 0$ has $x = 2$ as one of its solutions, then c must equal
(1) 1
(2) 2
(3) -1
(4) -2

18 Which statement is *not* true for any given parallelogram $ABCD$?
(1) $\overline{AB} \cong \overline{DC}$
(2) $\angle A \cong \angle C$
(3) $\overline{AC} \perp \overline{DB}$
(4) $m\angle B + m\angle C = 180$

19 If the measures of the three angles of a triangle are represented by x, $2x - 20$, and $3x - 10$, then the triangle is
(1) right
(2) obtuse
(3) isosceles
(4) equilateral

20 Which is an equation of a line that is perpendicular to the line whose equation is $y = -\frac{1}{2}x + 10$?
(1) $y = 2x + 3$
(2) $y = -2x + 3$
(3) $y = \frac{1}{2}x + 3$
(4) $y = -\frac{1}{2}x + 3$

21 Using the accompanying table, which is the solution set for $y^2 = 9$?

•	3	5	7	9
3	9	5	1	7
5	5	5	5	5
7	1	5	9	3
9	7	5	3	1

(1) {3}
(2) {7}
(3) {3,7}
(4) {3,7,9}

22 Given the true statements: "All tigers are animals" and "Max is not a tiger." It follows that
(1) Max is an animal
(2) Max is not an animal
(3) Max is a cat
(4) no conclusion can be reached

23 Which statement is logically equivalent to $\sim a \rightarrow b$?
(1) $a \rightarrow \sim b$
(2) $\sim b \rightarrow a$
(3) $b \rightarrow \sim a$
(4) $\sim b \rightarrow \sim a$

24 What is the negation of the statement, "Some students do not do their homework"?
(1) Some students do their homework.
(2) All students do not do their homework.
(3) All students do their homework.
(4) No student does his homework.

25 What is the length of the segment that joins two points whose coordinates are $(4,-1)$ and $(7,5)$?
(1) 5
(2) $\sqrt{13}$
(3) $\sqrt{29}$
(4) $\sqrt{45}$

26 Point P is on line $\overleftrightarrow{AB}$. The locus of the centers of all circles of radius 5 which pass through point P is
(1) a circle of radius 5 with center at P
(2) a line passing through P and perpendicular to $\overleftrightarrow{AB}$
(3) two lines, both perpendicular to $\overleftrightarrow{AB}$ and 5 units on either side of P
(4) two lines parallel to $\overleftrightarrow{AB}$, one 5 units above $\overleftrightarrow{AB}$ and the other 5 units below $\overleftrightarrow{AB}$

27 Which pair of triangles *must* be similar?
(1) two right triangles
(2) two obtuse triangles
(3) two scalene triangles with congruent bases
(4) two isosceles triangles with congruent vertex angles

28 Which statement is always true?
(1) The diagonals of a parallelogram are congruent.
(2) The diagonals of a parallelogram are perpendicular.
(3) The diagonals of a parallelogram bisect the angles of the parallelogram.
(4) The diagonals of a parallelogram bisect each other.

29 For the graph of which equation is $x = 2$ an equation of the axis of symmetry?
(1) $3x^2 + 6x - 8 = y$
(2) $x^2 + 2x - 3 = y$
(3) $x^2 - 4x - 6 = y$
(4) $4x^2 - 2x + 10 = y$

30 Which set of numbers could represent the lengths of the sides of a right triangle?
(1) {5,7,8}
(2) {7,8,12}
(3) {7,9,11}
(4) {8,15,17}

31 How many combinations can be made from 7 objects taken 4 at a time?
(1) 35
(2) 42
(3) 210
(4) 840

32 Which property is *not* necessary for a system to be a group?
(1) associative
(2) inverse
(3) closure
(4) distributive

33 Which is an equation of the circle whose center is at (0,0) and which passes through the point (3,4)?
(1) $x^2 + y^2 = 5$
(2) $(x - 3)^2 + (y - 4)^2 = 5$
(3) $x^2 + y^2 = 25$
(4) $(x - 3)^2 + (y - 4)^2 = 25$

34 Which is a solution for the system of equations $y = 2x - 15$ and $y = x^2 - 6x$?
(1) (3,−9)
(2) (0,0)
(3) (5,5)
(4) (6,0)

Directions (35): Leave all construction lines on the answer sheet.

35 *On the answer sheet*, construct a line perpendicular to $\overleftrightarrow{APB}$ through P.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Show all work unless otherwise directed.

36 Given $(C_4,+,\cdot)$, where $C_4 = \{0,1,2,3\}$, $+$ is addition clock 4, and $\cdot$ is multiplication clock 4.

a Construct an addition table and a multiplication table for C_4. [4]

b Find all elements which do not have a multiplicative inverse. [2]

c Which statement is true?

(1) $(C_4,+)$ and $(C_4,\cdot)$ are both groups.
(2) Only $(C_4,+)$ is a group.
(3) Only $(C_4,\cdot)$ is a group.
(4) Neither $(C_4,+)$ nor $(C_4,\cdot)$ is a group. [2]

d Give a reason why $(C_4,+,\cdot)$ is not a field. [2]

37 On an examination a student is to select any 4 out of 9 problems. All of the problems are of equal difficulty. The examination contains 1 geometry, 3 logic, 1 locus, and 4 probability problems.

a How many 4-problem selections could be made? [2]

b How many of those selections will contain 1 logic, 1 locus, and 2 probability problems? [4]

c What is the probability that a 4-problem selection contains 1 logic, 1 locus, and 2 probability problems? [2]

d What is the probability that a 4-problem selection will contain all logic problems? [2]

38 *a* Draw the graph of the equation $y = \frac{1}{2}x^2 - 4x + 4$ including all values of x such that $0 \leq x \leq 8$. [6]

b Write the coordinates of the turning point. [2]

c Between which pair of consecutive integers does one root of the equation $\frac{1}{2}x^2 - 4x + 4 = 0$ lie?

(1) 0 and 1 (3) 2 and 3
(2) 1 and 2 (4) -1 and 0 [2]

39 Triangle ABC has vertices $A(-3,-4)$, $B(-1,7)$, and $C(3,5)$. Find the area of $\triangle ABC$. [10]

40 In right triangle ABC, $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$. The length of $\overline{DB}$ is 6 units longer than the length of $\overline{AD}$.

a If $AD = x$, express the length of $\overline{DB}$ in terms of x. [1]

b If $CD = 4$, write an equation in terms of x to find AD. [3]

c Find AD by solving the equation written in part *b*. [4]

d Find the length of $\overline{AC}$ in radical form. [2]

Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Show all work unless otherwise directed.

41 Given:

If the radio is on and the television works, then there is electricity.
If the lights go out, then there is no electricity.
The lights go out.
The radio is on.

Let R represent: "The radio is on."
Let T represent: "The television works."
Let E represent: "There is electricity."
Let L represent: "The lights go out."

Using R, T, E, and L, prove: "The television does not work." [10]

42 Quadrilateral $TRAP$ has vertices $T(0,0)$, $R(0,5)$, $A(9,8)$, and $P(12,4)$. Prove by coordinate geometry that quadrilateral $TRAP$ is an isosceles trapezoid. [10]

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REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH — COURSE II

Monday, June 18, 1984 — 1:15 to 4:15 p.m., only

Part I Score	
Part II Score	
Part III Score	
Total Score	
Rater's Initials:	

ANSWER SHEET

Pupil ..... Teacher .....

School ..... Grade .....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 .....
2	12	22	32 .....
3	13	23	33 .....
4	14	24	34 .....
5	15	25	35 Answer question 35 on the other side of this sheet.
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Tear Here

35

Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

_____

Signature

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Monday, June 18, 1984 — 1:15 to 4:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 15–34, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 10

(2) $\frac{2}{6}$

(3) 40

(4) 50

(5) 7

(6) 20

(7) 6

(8) $3\sqrt{2}$ *or* $\sqrt{18}$

(9) 40

(10) 48

(11) 50

(12) 60

(13) -6

(14) 6

(15) 1

(16) 1

(17) 2

(18) 3

(19) 2

(20) 1

(21) 3

(22) 4

(23) 2

(24) 3

(25) 4

(26) 1

(27) 4

(28) 4

(29) 3

(30) 4

(31) 1

(32) 4

(33) 3

(34) 1

(35) construction

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a*

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

•	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

[4]

b 0,2 [1,1]

c 2 [2]

(37) *a* 126 [2]

b 18 [4]

c $\frac{18}{126}$ [2]

d 0 [2]

(38) *b* (4,−4) [2]

c 2 [2]

(39) 24 [10]

(40) *a* $x + 6$ [1]

b $x(x + 6) = 4^2$ [3]

c 2 [4]

d $2\sqrt{5}$ *or* $\sqrt{20}$ [2]