HIGH SCHOOL MATHEMATICS: COURSE III–JANUARY 1982 (1)

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers on a separate sheet. Where applicable, answers may be left in terms of π or in radical form.

1. Express $100°$ in radian measure.
2. Solve for x: $\frac{1}{2x} = \frac{1}{x} - 2$
3. Find the value of $\sin 750°$.
4. Evaluate: $\sum_{k=2}^{4} (k^2 - 1)$
5. Express in simplest form: $\frac{3 - \frac{3}{x}}{x - 1}$
6. In triangle ABC, $a = 2$, $b = 10$, and $\sin A = \frac{1}{6}$. Find $\sin B$.
7. If $\cot A = \frac{3}{4}$ and $\sin A > 0$, find $\sec A$.
8. Solve for x in terms of a and b: $\log_b x = a$
9. If $f(x) = \sin 2x$, find $f\left(\frac{\pi}{4}\right)$.
10. Point $P(-1, -5)$ is reflected over the line $y = -x$. What are the coordinates of P', the image of P?
11. Find the value of $\sin 65° 23'$ to *four decimal places.*
12. Chords $\overline{AB}$ and $\overline{CD}$ of circle O intersect at E. If $AE = EB = 4$ and $CE = 8$, find ED.
13. Express $\tan(-150°)$ as a function of a positive acute angle.
14. In a circle of radius 6, find the length of the arc intercepted by a central angle of 2 radians.
15. Find the numerical value of $8^{\frac{2}{3}} + 4^0$.
16. If the probability of a team's winning is $\frac{2}{3}$ and the probability of losing is $\frac{1}{3}$, what is the probability that the team will win exactly 1 of 4 games?
17. What is the period of the graph of $y = \frac{1}{2} \cos 2x$?
18. Find the positive value of $\sin \frac{1}{2}x$ if $\cos x = 0.02$.
19. A translation maps $P(4, -4)$ onto $P'(3, 0)$. Find the coordinates of Q', the image of $Q(3, 2)$, under the same translation.

Directions (20–35): For *each* question chosen, write on a separate sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

20. When $\sqrt{-3}$ is subtracted from $\sqrt{-12}$, the difference is
    (1) $i\sqrt{3}$ (2) $-i\sqrt{3}$ (3) $3i\sqrt{3}$ (4) $-3i\sqrt{3}$

21. The solution set of $3^{2x-1} = 3^{x^2}$ is
(1) $\{1\}$ (2) $\{-1\}$ (3) $\{1, -1\}$ (4) $\{\ \}$

22. The square of $(2 - 2i)$ is
(1) 0 (2) $-8i$ (3) $4 - 4i$ (4) 4

23. If $\theta = \text{Arc cos}\left(\frac{1}{2}\right)$, the value of $\sin\theta$ is
(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{\sqrt{3}}{2}$

24. If $\log 2 = A$ and $\log 3 = B$, then $\log 6$ is equal to
(1) $A + B$ (2) $A - B$ (3) AB (4) $\frac{A}{B}$

25. The expression $(\tan\theta)(\csc\theta)$ is equivalent to
(1) $\sin\theta$ (2) $\cos\theta$ (3) $\csc\theta$ (4) $\sec\theta$

26. *All* isosceles trapezoids have
(1) point symmetry, only
(2) line symmetry, only
(3) both point and line symmetry
(4) neither point nor line symmetry

27. If $\sin\theta$ and $\tan\theta$ have opposite signs, in which quadrants may angle θ lie?
(1) I and II (2) II and III (3) I and III (4) II and IV

28. Which property is *not* preserved under a dilation?
(1) distance (2) orientation (3) collinearity (4) angle measurement

29. If $\tan(A - 30) = \cot A$, the number of degrees in the measure of angle A is
(1) 30 (2) 45 (3) 60 (4) 90

30. What is the domain of the function $f(x) = \sqrt{x-1}$?
(1) $\{x|x \geq 1\}$ (2) $\{x|x \geq 2\}$ (3) $\{x|x \leq 1\}$ (4) $\{x|x \leq -2\}$

31. If l and m are parallel lines, then $r_l \circ r_m \overline{AB}$ is equivalent to a
(1) rotation (2) dilation (3) translation (4) glide-reflection

32. What approximate percentage of the scores of a normal distribution would be expected to fall within two standard deviations from the mean?
(1) 2.5% (2) 34% (3) 68% (4) 95%

33. In the interval $0° \leq \theta \leq 360°$, how many values of θ satisfy $\tan^2\theta - 1 = 0$?
(1) 1 (2) 2 (3) 3 (4) 4

34. In triangle ABC, $a = 2$, $b = 4$, and $m\angle C = 60$. What is the value of c?
(1) $2\sqrt{7}$ (2) 2 (3) $2\sqrt{3}$ (4) $4\sqrt{7}$

35. The fourth term in the expansion $(a - 3b)^5$ is
(1) $270a^2b^3$ (2) $-270a^2b^3$ (3) $90a^2b^3$ (4) $-90a^2b^3$

Part II

Answer four questions from this part. Show all work unless otherwise directed.

36. *a.* On the same set of axes, sketch the graphs of $y = \tan x$ and $y = \frac{1}{2}\cos x$, as x varies from 0 to 2π radians. [8]

b. State the number of values of x in the interval $0 \leq x \leq 2\pi$ that satisfy the equation

$$\tan x = \tfrac{1}{2} \cos x. \quad [2]$$

37. *a.* Find all values of θ in the interval $0° \leq \theta \leq 360°$ which satisfy the equation

$$2 \sin^2 \theta - 3 \sin \theta + 1 = 0. \quad [4]$$

b. For all values of θ for which the expressions are defined, prove that the following is an identity:

$$\frac{\cos \theta + \cot \theta}{\cos \theta \cot \theta} = \tan \theta + \sec \theta \quad [6]$$

38. *a.* Solve the equation $x^2 - 2x + 5 = 0$ and express the roots in the form $a + bi$. [5]
b. Using logarithms, solve for x to the *nearest tenth.*

$$2^x = 5 \quad [5]$$

39. A high school football team scored the following number of points during the ten-game season:

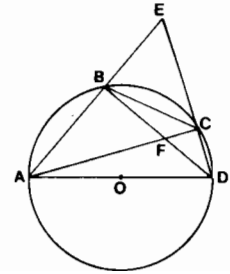
19, 20, 21, 27, 29, 29, 34, 40, 40, 41

a. What is the median? [1]
b. What is the mean? [2]
c. Find the standard deviation of these scores to the *nearest tenth.* [7]

40. *a.* Two sides of a triangular plot measure 30 meters and 18 meters, respectively. If the angle opposite the 30-meter side measures 58°, find, to the *nearest degree,* the measure of the angle opposite the 18-meter side. [6]
b. Using the answer to part *a*, find the area of the triangle to the *nearest square meter.* [4]

41. The coordinates of the endpoints of line segment $\overline{AB}$ are $A(4, 1)$ and $B(5, 4)$.
a. Graph $\overline{AB}$. [2]
b. Graph $\overline{A'B'}$, the image of $\overline{AB}$, after a reflection over the line $y = x$. [2]
c. Graph $\overline{A''B''}$, the image of $\overline{A'B'}$, after the transformation $(x, y) \rightarrow (x - 5, y - 5)$. [2]
d. Graph $\overline{A'''B'''}$, the image of $\overline{A''B''}$, after a reflection through the origin. [2]
e. Write a translation which will map $\overline{A'B'}$ onto $\overline{B'''A'''}$. [2]

42. In the accompanying figure, quadrilateral $ABCD$ is inscribed in circle O. Diagonals $\overline{AC}$ and $\overline{BD}$ meet at F, and $\overline{AD}$ is a diameter. Chords $\overline{AB}$ and $\overline{DC}$ are extended to meet at E. B is the midpoint of $\overset{\frown}{AC}$ and $m\overset{\frown}{AB} : m\overset{\frown}{CD} = 4 : 1$.
Find:



a. $m\overset{\frown}{CD}$ [2]
b. $m\angle BDA$ [2]
c. $m\angle BFC$ [2]
d. $m\angle E$ [2]
e. $m\angle EBD$ [2]