

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Friday, January 27, 1984 — 9:15 a.m. to 12:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly, and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The "Reference Tables for Mathematics" and a formula sheet which you may need to answer some questions in this examination are included in this booklet following page 6.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.

- Express 260° in radian measure.
- Find the value of $\sin^2 \frac{\pi}{3}$.
- If $f(x) = x + x^{-1}$, find the value of $f(4)$.

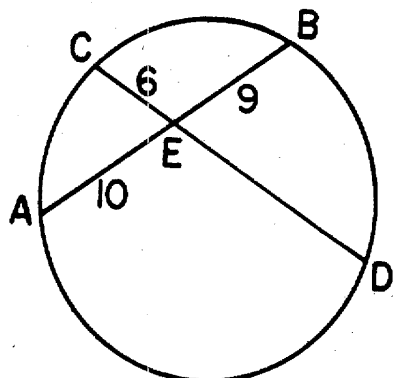
4 Express in simplest form:
$$\frac{x - \frac{1}{x}}{1 - \frac{1}{x}}$$

5 Evaluate:
$$\sum_{k=1}^3 \frac{1}{2}(k^2)$$

6 Express $\cos(-220^\circ)$ as a function of a positive acute angle.

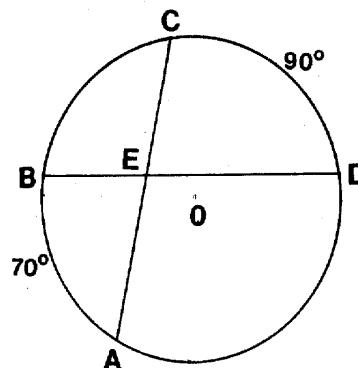
7 Find x if $\log_9 x = \frac{3}{2}$.

8 In the accompanying diagram, \overline{AB} and \overline{CD} are chords of the circle and intersect at E . If $AE = 10$, $EB = 9$, and $CE = 6$, find DE .



9 In a circle, a central angle of 3 radians intercepts an arc of length 12. What is the length of the radius of the circle?

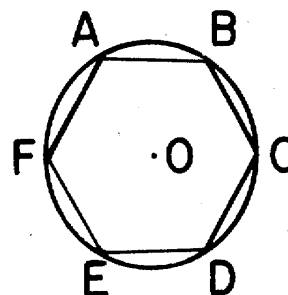
10 In the accompanying diagram, \overline{AC} and \overline{BD} are chords of circle O and intersect at E . If $m\widehat{AB} = 70$ and $m\widehat{CD} = 90$, find $m\angle BEA$.



11 If the solution set of $x^2 + px + q = 0$ is $\{1 + i, 1 - i\}$, find the value of p .

12 Find the value of $\log 58.43$ to four decimal places.

13 In the accompanying diagram, regular hexagon $ABCDEF$ is inscribed in circle O . With O as the center of rotation find $\text{Rot}_{(-120^\circ)} \circ \text{Rot}_{(240^\circ)}(A)$.



14 In $\triangle ABC$, $\sin A = \frac{1}{3}$, $\sin B = \frac{1}{5}$, and $b = 6$. Find side a .

15 If $4^{2x} = 2^{3x+2}$, find the value of x .

16 Express in terms of i the sum of $\sqrt{-25} + 2\sqrt{-36}$.

17 Find $\tan (\text{Arc cos } \frac{3}{5})$.

18 For what value of k are the roots of $2x^2 - 8x + k = 0$ equal?

Directions (19–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

19 Which letter has both vertical and horizontal line symmetry?

- (1) A (3) T
(2) M (4) X

20 Which is *not* an element in the range of the function $y = \cos x$?

- (1) 1 (3) $\frac{1}{2}$
(2) 2 (4) $-\frac{1}{2}$

21 The expression $(2 + i)^2$ is equivalent to

- (1) $3 + 4i$ (3) 3
(2) $5 + 4i$ (4) 5

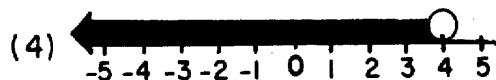
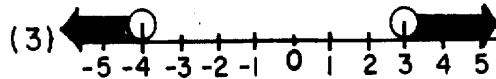
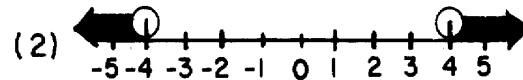
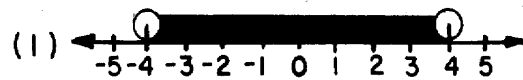
22 The fifth term in the expansion of $(2x - y)^6$ is

- (1) $240x^2y^4$ (3) $60x^2y^4$
(2) $-240x^2y^4$ (4) $-60x^2y^4$

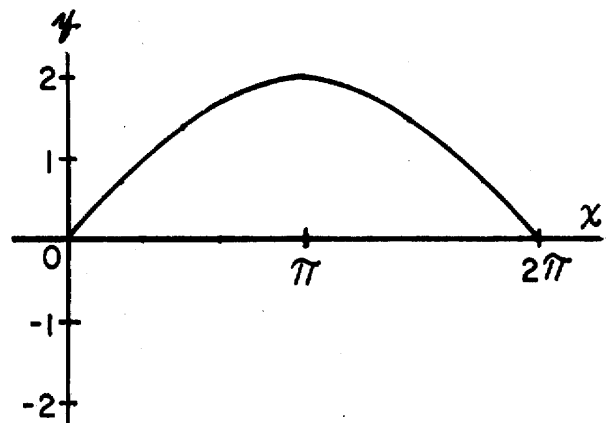
23 On a test, the mean score is 25 and the standard deviation is 2.3. Which score could be expected to occur *less than 5%* of the time?

- (1) 20 (3) 23
(2) 28 (4) 24

24 Which is the graph of $(-x > 4) \vee (2x - 1 > 7)$?



25 Which is the equation of the graph shown below?



- (1) $y = 2 \sin \frac{1}{2}x$ (3) $y = 2 \cos \frac{1}{2}x$
(2) $y = \frac{1}{2} \sin 2x$ (4) $y = \frac{1}{2} \cos 2x$

26 The equation $x = \sqrt{3x + 4}$ has

- (1) 4 and -1 as solutions
(2) 4 as its only solution
(3) -1 as its only solution
(4) no solutions

27 If a fair coin is tossed four times, what is the probability of obtaining *at most one* head?

- (1) $\frac{1}{16}$ (3) $\frac{5}{16}$
(2) $\frac{4}{16}$ (4) $\frac{11}{16}$

28 If the sum of a number n and ten times its reciprocal is 7, then a value of n may be

- (1) $\frac{1}{2}$ (3) 3
 (2) 2 (4) 4

29 The expression $2 \sin^2 A + \cos 2A$ is equivalent to

- (1) 1 (3) $\sin^2 A$
 (2) 2 (4) $-\sin^2 A$

30 The amplitude of the graph of the equation $y = 4 \sin 2x$ is

- (1) 1 (3) $\frac{1}{2}$
 (2) 2 (4) 4

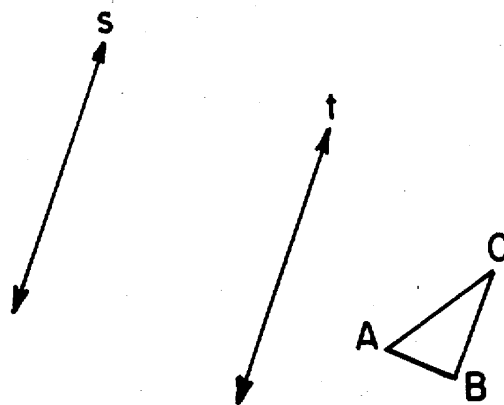
31 The expression $\frac{1}{3} \log a - 3 \log b$ is equivalent to

- (1) $\log(\sqrt[3]{a} - b^3)$ (3) $\log \frac{\sqrt[3]{a}}{b^3}$
 (2) $\log \frac{a}{3b^3}$ (4) $\log \frac{\sqrt[3]{a}}{3b}$

32 The expression $\frac{1}{2 - \sqrt{3}}$ is equivalent to

- (1) $2 + \sqrt{3}$ (3) $\frac{2 + \sqrt{3}}{-1}$
 (2) $2 - \sqrt{3}$ (4) $\frac{2 - \sqrt{3}}{-1}$

33 In the accompanying diagram, line s is parallel to line t . Which is equivalent to the composition of line reflections $r_s \circ r_t$ ($\triangle ABC$)?

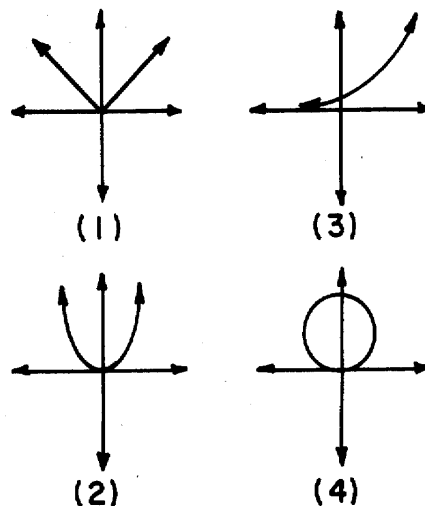


- (1) a rotation (3) a translation
 (2) a line reflection (4) a glide-reflection

34 In the interval $0^\circ \leq \theta \leq 360^\circ$, how many values of θ satisfy the equation

- $\tan^2 \theta - 3 \tan \theta + 2 = 0$?
 (1) 1 (3) 3
 (2) 2 (4) 4

35 Which graph has point symmetry?



Answers to the following questions are to be written on paper provided by the school.

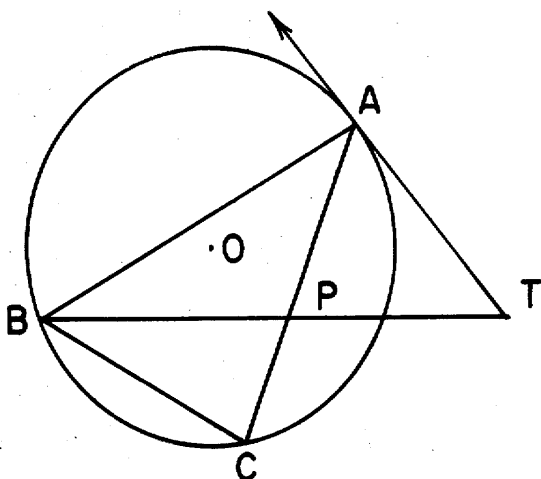
Part II

Answer four questions from this part. Show all work unless otherwise directed.

36 a On the same set of axes, sketch and label the graphs of $y = 2 \cos \frac{1}{2}x$ and $y = -1$ for the values of x in the interval $0 \leq x \leq 2\pi$. [6,2]

b State the number of values of x in the interval $0 \leq x \leq 2\pi$ that satisfy the equation $2 \cos \frac{1}{2}x = -1$. [2]

37 In the accompanying figure, $\triangle ABC$ is inscribed in circle O , \overline{BT} bisects $\angle CBA$, \overline{TA} is tangent to circle O , and $m\angle BAC : m\angle CBA : m\angle ACB = 2:3:4$.



Find:

- a $m\angle BAC$ [2]
- b $m\widehat{BC}$ [2]
- c $m\angle CPT$ [2]
- d $m\angle PAT$ [2]
- e $m\angle T$ [2]

38 In $\triangle ABC$, $a = 10$, $b = 12$, and the measure of angle C is $41^\circ 30'$.

- a Find the length of side c to the nearest integer. [6]
- b Find the area of $\triangle ABC$ to the nearest tenth. [4]

39 a Solve for x to the nearest tenth:

$$\log_x 5 = 3 \quad [5]$$

b Express in simplest form:

$$\frac{3a + 1}{a^2 - 1} - \frac{1}{a + 1} \quad [5]$$

40 Triangle ABC has coordinates $A(1,1)$, $B(5,1)$, and $C(4,3)$. Given the transformations T , U , and W described below:

$$T: (x,y) \rightarrow (x,-y)$$

$$U: (x,y) \rightarrow (x-6,y+6)$$

$$W: (x,y) \rightarrow (-2x,-2y)$$

- a Graph $\triangle ABC$ and graph and state the coordinates of its image $\triangle A'B'C'$, after transformation T . [3]
- b Graph and state the coordinates of $\triangle A''B''C''$, the image of $\triangle ABC$ after transformation U . [2]
- c Graph and state the coordinates of $\triangle A'''B'''C'''$, the image of $\triangle ABC$ after transformation W . [3]
- d Which transformation, T , U , or W , is not an isometry? [1]
- e Which transformation, T , U , or W , does not preserve orientation? [1]

➡ GO RIGHT ON TO THE NEXT PAGE.

- 41 Using the following set of data, find
a the mean [3]
b the standard deviation to the nearest tenth [7]

x_i measure	f_i frequency
50	4
58	4
62	3
64	6
65	2
68	1

- 42 a Find all values of θ in the interval $0^\circ \leq \theta \leq 360^\circ$ which satisfy the equation $2 \cos \theta + 1 = \sec \theta$. [5]

- b Prove the identity:
 $\tan A + \cot A = \sec A \csc A$ [5]

Formulas

Pythagorean and Quotient Identities

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 & \tan A &= \frac{\sin A}{\cos A} \\ \tan^2 A + 1 &= \sec^2 A & \cot A &= \frac{\cos A}{\sin A} \\ \cot^2 A + 1 &= \csc^2 A\end{aligned}$$

Functions of the Sum of Two Angles

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B}\end{aligned}$$

Functions of the Difference of Two Angles

$$\begin{aligned}\sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}\end{aligned}$$

Functions of the Double Angle

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ \cos 2A &= 2 \cos^2 A - 1 \\ \cos 2A &= 1 - 2 \sin^2 A \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}\end{aligned}$$

Functions of the Half Angle

$$\begin{aligned}\sin \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{2}} \\ \cos \frac{1}{2}A &= \pm \sqrt{\frac{1 + \cos A}{2}} \\ \tan \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}\end{aligned}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of Triangle

$$K = \frac{1}{2}ab \sin C$$

Standard Deviation

$$S.D. = \sqrt{\frac{1}{n} \sum_{i=1}^n (\bar{x} - x_i)^2}$$

(B) Values of Trigonometric Functions

Angle	Sin	Cos	Tan	Cot	
24° 00'	.4067	.9135	.4452	2.2460	66° 00'
10	.4094	.9124	.4487	2.2286	50
20	.4120	.9112	.4522	2.2113	40
30	.4147	.9100	.4557	2.1943	30
40	.4173	.9088	.4592	2.1775	20
50	.4200	.9075	.4628	2.1609	10
25° 00'	.4226	.9063	.4663	2.1445	65° 00'
10	.4253	.9051	.4699	2.1283	50
20	.4279	.9038	.4734	2.1123	40
30	.4305	.9026	.4770	2.0965	30
40	.4331	.9013	.4806	2.0809	20
50	.4358	.9001	.4841	2.0655	10
26° 00'	.4384	.8988	.4877	2.0503	64° 00'
10	.4410	.8975	.4913	2.0353	50
20	.4436	.8962	.4950	2.0204	40
30	.4462	.8949	.4986	2.0057	30
40	.4488	.8936	.5022	1.9912	20
50	.4514	.8923	.5059	1.9768	10
27° 00'	.4540	.8910	.5095	1.9626	63° 00'
10	.4566	.8897	.5132	1.9486	50
20	.4592	.8884	.5169	1.9347	40
30	.4617	.8870	.5206	1.9210	30
40	.4643	.8857	.5243	1.9074	20
50	.4669	.8843	.5280	1.8940	10
28° 00'	.4695	.8829	.5317	1.8807	62° 00'
10	.4720	.8816	.5354	1.8676	50
20	.4746	.8802	.5392	1.8546	40
30	.4772	.8788	.5430	1.8418	30
40	.4797	.8774	.5467	1.8291	20
50	.4823	.8760	.5505	1.8165	10
29° 00'	.4848	.8746	.5543	1.8040	61° 00'
10	.4874	.8732	.5581	1.7917	50
20	.4899	.8718	.5619	1.7796	40
30	.4924	.8704	.5658	1.7675	30
40	.4950	.8689	.5696	1.7556	20
50	.4975	.8675	.5735	1.7437	10
30° 00'	.5000	.8660	.5774	1.7321	60° 00'
10	.5025	.8646	.5812	1.7205	50
20	.5050	.8631	.5851	1.7090	40
30	.5075	.8616	.5890	1.6977	30
40	.5100	.8601	.5930	1.6864	20
50	.5125	.8587	.5969	1.6753	10
31° 00'	.5150	.8572	.6009	1.6643	59° 00'
10	.5175	.8557	.6048	1.6534	50
20	.5200	.8542	.6088	1.6426	40
30	.5225	.8526	.6128	1.6319	30
40	.5250	.8511	.6168	1.6212	20
50	.5275	.8496	.6208	1.6107	10
32° 00'	.5299	.8480	.6249	1.6003	58° 00'
10	.5324	.8465	.6289	1.5900	50
20	.5348	.8450	.6330	1.5798	40
30	.5373	.8434	.6371	1.5697	30
40	.5398	.8418	.6412	1.5597	20
50	.5422	.8403	.6453	1.5497	10
33° 00'	.5446	.8387	.6494	1.5399	57° 00'
10	.5471	.8371	.6536	1.5301	50
20	.5495	.8355	.6577	1.5204	40
30	.5519	.8339	.6619	1.5108	30
40	.5544	.8323	.6661	1.5013	20
50	.5568	.8307	.6703	1.4919	10
34° 00'	.5592	.8290	.6745	1.4826	56° 00'
10	.5616	.8274	.6787	1.4733	50
20	.5640	.8258	.6830	1.4641	40
30	.5664	.8241	.6873	1.4550	30
40	.5688	.8225	.6916	1.4460	20
50	.5712	.8208	.6959	1.4370	10
35° 00'	.5736	.8192	.7002	1.4281	55° 00'
10	.5760	.8175	.7046	1.4193	50
20	.5783	.8158	.7089	1.4106	40
30	.5807	.8141	.7133	1.4019	30
40	.5831	.8124	.7177	1.3934	20
50	.5854	.8107	.7221	1.3848	10
36° 00'	.5878	.8090	.7265	1.3764	54° 00'
	Cos	Sin	Cot	Tan	Angle

Angle	Sin	Cos	Tan	Cot	
36° 00'	.5878	.8090	.7265	1.3764	54° 00'
10	.5901	.8073	.7310	1.3680	50
20	.5925	.8056	.7355	1.3597	40
30	.5948	.8039	.7400	1.3514	30
40	.5972	.8021	.7445	1.3432	20
50	.5995	.8004	.7490	1.3351	10
37° 00'	.6018	.7986	.7536	1.3270	53° 00'
10	.6041	.7969	.7581	1.3190	50
20	.6065	.7951	.7627	1.3111	40
30	.6088	.7934	.7673	1.3032	30
40	.6111	.7916	.7720	1.2954	20
50	.6134	.7898	.7766	1.2876	10
38° 00'	.6157	.7880	.7813	1.2799	52° 00'
10	.6180	.7862	.7860	1.2723	50
20	.6202	.7844	.7907	1.2647	40
30	.6225	.7826	.7954	1.2572	30
40	.6248	.7808	.8002	1.2497	20
50	.6271	.7790	.8050	1.2423	10
39° 00'	.6293	.7771	.8098	1.2349	51° 00'
10	.6316	.7753	.8146	1.2276	50
20	.6338	.7735	.8195	1.2203	40
30	.6361	.7716	.8243	1.2131	30
40	.6383	.7698	.8292	1.2059	20
50	.6406	.7679	.8342	1.1988	10
40° 00'	.6428	.7660	.8391	1.1918	50° 00'
10	.6450	.7642	.8441	1.1847	50
20	.6472	.7623	.8491	1.1778	40
30	.6494	.7604	.8541	1.1708	30
40	.6517	.7585	.8591	1.1640	20
50	.6539	.7566	.8642	1.1571	10
41° 00'	.6561	.7547	.8693	1.1504	49° 00'
10	.6583	.7528	.8744	1.1436	50
20	.6604	.7509	.8796	1.1369	40
30	.6626	.7490	.8847	1.1303	30
40	.6648	.7470	.8899	1.1237	20
50	.6670	.7451	.8952	1.1171	10
42° 00'	.6691	.7431	.9004	1.1106	48° 00'
10	.6713	.7412	.9057	1.1041	50
20	.6734	.7392	.9110	1.0977	40
30	.6756	.7373	.9163	1.0913	30
40	.6777	.7353	.9217	1.0850	20
50	.6799	.7333	.9271	1.0786	10
43° 00'	.6820	.7314	.9325	1.0724	47° 00'
10	.6841	.7294	.9380	1.0661	50
20	.6862	.7274	.9435	1.0599	40
30	.6884	.7254	.9490	1.0538	30
40	.6905	.7234	.9545	1.0477	20
50	.6926	.7214	.9601	1.0416	10
44° 00'	.6947	.7193	.9657	1.0355	46° 00'
10	.6967	.7173	.9713	1.0295	50
20	.6988	.7153	.9770	1.0235	40
30	.7009	.7133	.9827	1.0176	30
40	.7030	.7112	.9884	1.0117	20
50	.7050	.7092	.9942	1.0058	10
45° 00'	.7071	.7071	1.0000	1.0000	45° 00'
	Cos	Sin	Cot	Tan	Angle

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH — COURSE III

Friday, January 27, 1984 — 9:15 a.m. to 12:15 p.m., only

Part I Score:

Rater's Initials:

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ANSWER SHEET

Pupil.....Teacher.....

School.....Grade.....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

- | | | | |
|----------|----------|----------|----------|
| 1 | 11 | 21 | 31 |
| 2 | 12 | 22 | 32 |
| 3 | 13 | 23 | 33 |
| 4 | 14 | 24 | 34 |
| 5 | 15 | 25 | 35 |
| 6 | 16 | 26 | |
| 7 | 17 | 27 | |
| 8 | 18 | 28 | |
| 9 | 19 | 29 | |
| 10 | 20 | 30 | |

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Friday, January 27, 1984 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 19–35, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) $\frac{13\pi}{9}$	(11) -2	(21) 1	(31) 3
(2) $\frac{3}{4}$	(12) 1.7666	(22) 3	(32) 1
(3) $\frac{17}{4}$	(13) E	(23) 1	(33) 3
(4) $x + 1$	(14) 10	(24) 2	(34) 4
(5) 7	(15) 2	(25) 1	(35) 4
(6) $-\cos 40^\circ$ or $-\sin 50^\circ$	(16) $17i$	(26) 2	
(7) 27	(17) $\frac{4}{3}$	(27) 3	
(8) 15	(18) 8	(28) 2	
(9) 4	(19) 4	(29) 1	
(10) 80	(20) 2	(30) 4	

[OVER]

SEQUENTIAL MATH — COURSE III — *concluded*

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) b 1 [2]

(37) a 40 [2]

b 80 [2]

c 110 [2]

d 60 [2]

e 50 [2]

(38) a 8 [6]

b 39.8 [4]

(39) a 1.7 [5]

b $\frac{2}{a-1}$ [5]

(40) a $A'(1,-1)$, $B'(5,-1)$, $C'(4,-3)$ [3]

b $A''(-5,7)$, $B''(-1,7)$, $C''(-2,9)$ [2]

c $A'''(-2,-2)$, $B'''(-10,-2)$,

$C'''(-8,-6)$ [3]

d W [1]

e T [1]

(41) a 60 [3]

b 5.6 [7]

(42) a 60° , 180° , 300° [5]