

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, June 21, 2000 — 1:15 to 4:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 In $\triangle ABC$, $\sin A = 0.3$, $\sin B = 0.8$, and $b = 12$. Find the length of side a .

2 If $f(x) = \sin 2x + \cos x$, find $f\left(\frac{\pi}{2}\right)$.

3 An angle inscribed in a circle measures 80 degrees. What is the number of degrees in the intercepted arc?

4 In $\triangle ABC$, $a = 1.3$, $b = 2.4$, and $m\angle C = 30$. Find the area of $\triangle ABC$.

5 Solve for x : $9^{2x} = 27^{x+1}$

6 If $\sin A < 0$ and $\cot A > 0$, in which quadrant does the terminal side of $\angle A$ lie?

7 A translation maps $P(3,-2)$ to $P'(1,1)$. Under the same translation, find the coordinates of Q' , the image of $Q(-3,2)$.

8 Factor completely: $9x^3 - x$

9 Solve for all values of x : $|2x + 3| = 7$

10 If $g(x) = 36^x$, evaluate $g\left(-\frac{1}{2}\right)$.

11 Evaluate: $2 \sum_{n=1}^4 n^2$

12 Express in simplest form: $\frac{\frac{x-y}{y}}{\frac{1}{y} - \frac{1}{x}}$

13 Express $\sqrt{-2} + \sqrt{-18}$ as a monomial in terms of i .

14 In a circle whose radius is 2 centimeters, a central angle intercepts an arc of 6 centimeters. What is the number of radians in the central angle?

15 Determine the maximum number of triangles possible when $m\angle A = 150$, $a = 14$, and $b = 10$.

16 Solve for x : $x - 1 = \sqrt{2x+13}$

Directions (17–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

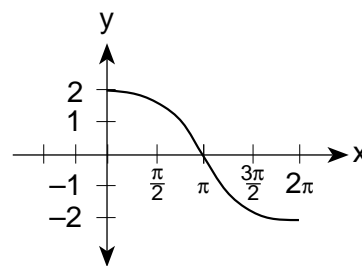
17 Which transformation is *not* an isometry?

- (1) dilation
- (2) rotation
- (3) reflection
- (4) translation

18 The expression $\sin \theta(\cot \theta - \csc \theta)$ is equivalent to

- (1) $\cos \theta - \sin^2 \theta$
- (2) $2 \cos \theta$
- (3) $-\sin \theta$
- (4) $\cos \theta - 1$

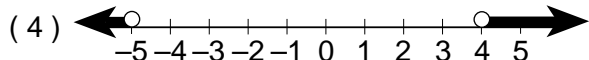
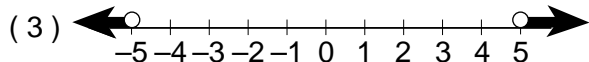
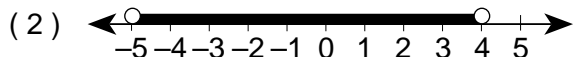
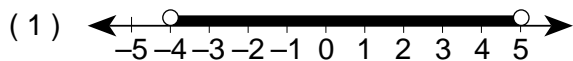
19 Which equation is sketched in the accompanying graph?



- (1) $y = \cos \frac{1}{2}x$
- (2) $y = \frac{1}{2} \cos x$
- (3) $y = 2 \cos \frac{1}{2}x$
- (4) $y = 2 \cos 2x$

- 20 As angle x increases from $\frac{\pi}{2}$ to π , the value of $\sin x$ will
- (1) increase from -1 to 0
 - (2) increase from 0 to 1
 - (3) decrease from 0 to -1
 - (4) decrease from 1 to 0

- 21 Which graph represents the solution set for the inequality $x^2 - x - 20 < 0$?



- 22 If the fraction $\frac{123}{10,000}$ is expressed in the form 1.23×10^n , the value of n is

- (1) -1
- (2) -2
- (3) -3
- (4) -4

- 23 If $f(x) = \sin(\text{Arc tan } x)$, the value of $f(1)$ is

- (1) $\sqrt{2}$
- (2) $\frac{\sqrt{2}}{2}$
- (3) $\frac{\sqrt{3}}{2}$
- (4) $\frac{\sqrt{3}}{3}$

- 24 A solution of the equation $\cos 2\theta + \sin 2\theta = -1$ is

- (1) 240°
- (2) 135°
- (3) 45°
- (4) -30°

- 25 In circle O , \overline{PA} and \overline{PB} are tangent to the circle from point P . If the ratio of the measure of major arc AB to the measure of minor arc AB is $5:1$, then $m\angle P$ is

- (1) 60
- (2) 90
- (3) 120
- (4) 180

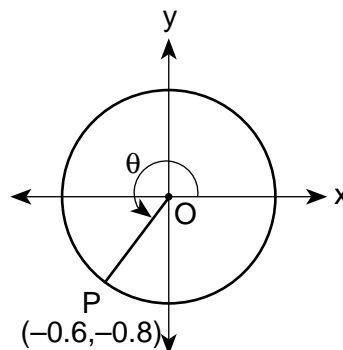
- 26 Which equation is *not* a function?

- (1) $3x^2 + 4y^2 = 12$
- (2) $y = 2 \cos x$
- (3) $y = 2^x$
- (4) $y = \log_2 x$

- 27 When the sum of $4 + 6i$ and $6 - 8i$ is graphed, in which quadrant does it lie?

- (1) I
- (2) II
- (3) III
- (4) IV

- 28 In the accompanying diagram, point $P(-0.6, -0.8)$ is on unit circle O .



What is the measure of angle θ to the nearest degree?

- (1) 143
- (2) 217
- (3) 225
- (4) 233

- 29 The expression $\log 12$ is equivalent to

- (1) $\log 6 + \log 6$
- (2) $\log 3 + 2 \log 2$
- (3) $\log 3 - 2 \log 2$
- (4) $\log 3 \cdot \log 4$

- 30 In the equation $x^2 - 7x + 2 = 0$, the sum of the roots exceeds the product of the roots by

- (1) 9
- (2) 5
- (3) -9
- (4) -5

- 31 What is the third term in the expansion of $(a - 3b)^4$?

- (1) $6a^2b^2$
- (2) $-6a^2b^2$
- (3) $54a^2b^2$
- (4) $-54a^2b^2$

- 32 The roots of the equation $2x^2 - 4x + k = 0$ are real and equal if k is equal to

- (1) -2
- (2) 2
- (3) -4
- (4) 4

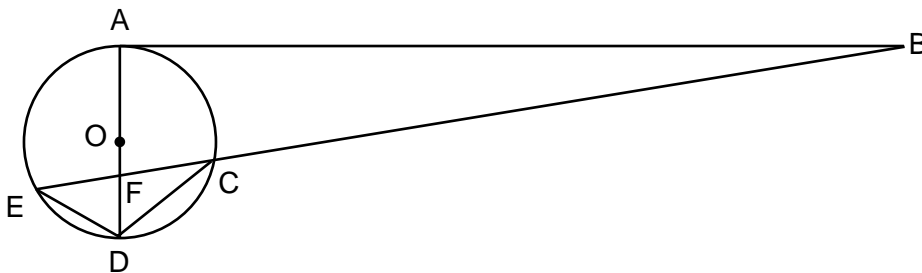
- 33 The expression $\cos(270^\circ - A)$ is equivalent to
- (1) $\cos A$ (3) $\sin A$
 (2) $-\cos A$ (4) $-\sin A$
- 34 The scores on a test approximate a normal distribution with a mean score of 72 and a standard deviation of 9. Approximately what percent of the students taking the test received a score greater than 90?
- (1) $2\frac{1}{2}\%$ (3) 10%
 (2) 5% (4) 16%
- 35 Mr. and Mrs. Douville have six children. What is the probability that there is *exactly one* female child? [Assume that $P(\text{male}) = P(\text{female})$.]
- (1) $\frac{1}{64}$ (3) $\frac{6}{64}$
 (2) $\frac{5}{64}$ (4) $\frac{32}{64}$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

- 36 In the accompanying diagram of circle O , tangent \overline{BA} , diameter \overline{AD} , secant \overline{BCE} , \overline{AD} intersects \overline{BE} at F , chords \overline{DE} and \overline{DC} are drawn, $m\angle AFB = 80$, and $m\widehat{AC} = 100$.



Find:

- a $m\angle BED$ [2]
 b $m\angle BAF$ [2]
 c $m\widehat{ED}$ [2]
 d $m\angle B$ [2]
 e $m\angle EDC$ [2]

GO RIGHT ON TO THE NEXT PAGE

- 37 a On the same set of axes, sketch and label the graphs of the equations $y = \sin \frac{1}{2}x$ and $y = -3 \cos 2x$ in the interval $-\pi \leq x \leq \pi$. [8]
- b Using the graphs drawn in part a, find the number of values of x in the interval $-\pi \leq x \leq \pi$ that satisfy the equation $\sin \frac{1}{2}x = -3 \cos 2x$. [2]
- 38 a Assume that in the United States $\frac{1}{5}$ of all cars are red. Suppose you are driving down the highway and you pass 6 cars.
- (1) What is the probability that *at most* one of the cars you pass is red? [3]
- (2) What is the probability that *at least* four of the cars you pass are red? [3]
- b The scores on a mathematics test are 42, 51, 58, 64, 70, 76, 76, 82, 84, 88, 88, 90, 94, 94, 94, and 97. For this set of data, find the standard deviation to the *nearest tenth*. [4]
- 39 Find all values of x in the interval $0 \leq x < 360^\circ$ that satisfy the equation $4 \cos^2 x - 5 \sin x - 5 = 0$. Express your answer to the *nearest ten minutes* or *nearest tenth of a degree*. [10]
- 40 a On the same set of axes, sketch and label the graphs of the equations $xy = 8$ and $x = 2^y$. [6]
- b On the same set of axes used in part a, sketch the reflection of $x = 2^y$ in the line $y = x$. Label it *b*. [3]
- c Write an equation of the graph drawn in part b. [1]
- 41 a Two forces of 50 pounds and 69 pounds act on a body to produce a resultant of 70 pounds. Find, to the *nearest tenth of a degree* or *nearest ten minutes*, the angle formed between the resultant and the smaller force. [6]
- b For all values of θ for which the expressions are defined, prove the following is an identity:
- $$(\cot \theta + \csc \theta)(1 - \cos \theta) = \sin \theta \quad [4]$$
- 42 a Solve for x and express your answer in simplest $a + bi$ form:
- $$x^2 - 10x = -41 \quad [6]$$
- b Express in simplest form:
- $$\frac{81 - x^2}{6x - 54} \div \frac{x^2 + 9x}{3x} \quad [4]$$

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Wednesday, June 21, 2000 — 1:15 to 4:15 p.m., only

| |
|--------------------------------|
| Part I Score |
| Part II Score |
| Total Score |
| Rater's Initials: |

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ANSWER SHEET

Pupil Sex: Male Female Grade

Teacher School

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

- | | | | |
|----------|----------|----------|----------|
| 1 | 11 | 21 | 31 |
| 2 | 12 | 22 | 32 |
| 3 | 13 | 23 | 33 |
| 4 | 14 | 24 | 34 |
| 5 | 15 | 25 | 35 |
| 6 | 16 | 26 | |
| 7 | 17 | 27 | |
| 8 | 18 | 28 | |
| 9 | 19 | 29 | |
| 10 | 20 | 30 | |

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

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Tear Here

FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

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SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 17–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

| | | | |
|-------------------------|-------------------|--------|--------|
| (1) 4.5 | (11) 60 | (21) 1 | (31) 3 |
| (2) 0 | (12) x | (22) 2 | (32) 2 |
| (3) 160 | (13) $4i\sqrt{2}$ | (23) 2 | (33) 4 |
| (4) 0.78 | (14) 3 | (24) 2 | (34) 1 |
| (5) 3 | (15) 1 | (25) 3 | (35) 3 |
| (6) III | (16) 6 | (26) 1 | |
| (7) (-5,5) | (17) 1 | (27) 4 | |
| (8) $x(3x + 1)(3x - 1)$ | (18) 4 | (28) 4 | |
| (9) -5,2 | (19) 3 | (29) 2 | |
| (10) $\frac{1}{6}$ | (20) 4 | (30) 2 | |

[OVER]

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics*, 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) a 40 [2]
 b 90 [2]
 c 60 [2]
 d 10 [2]
 e 110 [2]

(40) c $y = 2^x$ or $\log_2 y = x$ [1]

(41) a 67.9° or $67^\circ 50'$ [6]

(42) a $5 \pm 4i$ [6]

b $-\frac{1}{2}$ [4]

(37) b 4 [2]

(38) a (1) $\frac{10240}{15625}$ [3]

(2) $\frac{265}{15625}$ [3]

b 16.2 [4]

(39) $194.5^\circ, 270^\circ, 345.5^\circ$
or [10]
 $194^\circ 30', 270^\circ, 345^\circ 30'$