

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Tuesday, June 16, 1981 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The "Reference Tables for Mathematics" and a formula sheet which you may need to answer some questions in this examination are stapled in the center of this booklet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

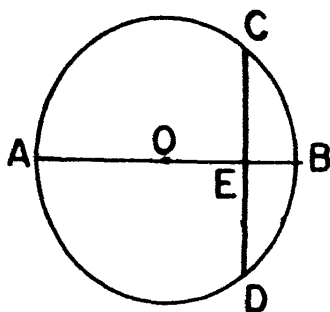
Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.

1 If $f(x) = 4x^{\frac{1}{2}}$, find $f(4)$.

2 Express the product of $(2 + \sqrt{-9})$ and $(3 - \sqrt{-16})$ in the form $a + bi$.

3 Express in simplest form:
$$\frac{n - \frac{1}{n}}{\frac{1 - n^2}{n}}$$

4 In the accompanying diagram of circle O , diameter $\overline{AB} \perp \overline{CD}$, and $CD = 14$. Find CE .



5 Find $\tan \left(\text{Arc sin } \frac{\sqrt{2}}{2} \right)$.

6 In a circle, a central angle of 2 radians intercepts an arc of length 12 centimeters. Find the length of the radius in centimeters.

7 Express $\sin 230^\circ$ as a function of a positive acute angle.

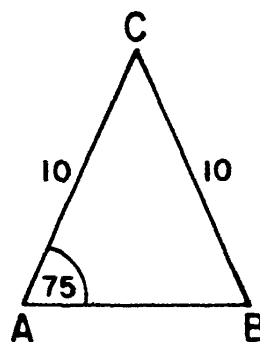
8 Express $\frac{7\pi}{10}$ radians in degree measure.

9 What is the image of the point $(2, -3)$ under a clockwise rotation of 90° (R_{-90°) about the origin?

10 If $\log_6 81 = x$, find x .

11 Find the value of $\sum_{k=1}^2 \sin \frac{k\pi}{2}$.

12 In the accompanying figure of $\triangle ABC$, $a = 10$, $b = 10$, and $m\angle A = 75^\circ$. Find the area of $\triangle ABC$.



13 Two tangents \overline{PA} and \overline{PB} are drawn to circle O from an external point P . If the measure of major arc \widehat{AB} is 250° , find the measure of $\angle P$.

14 Find the value of $\log 429.7$ correct to four decimal places.

15 In $\triangle RST$, $\sin R = 0.6$, $\sin S = 0.4$, and side $s = 16$. Find the length of side r .

16 Find the value of x in the interval $90^\circ < x < 180^\circ$ which satisfies the equation $\cos x - 2 \cos x \sin x = 0$.

17 If $\cos x = \frac{3}{5}$, what is the positive value of $\sin \frac{1}{2}x$?

18 If the probability of winning a game is $\frac{1}{4}$, find the probability of winning at least 3 games out of 4.

Directions (19–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

19 Which letter has vertical line symmetry?

- (1) A (3) N
(2) S (4) D

20 Which graph has line symmetry with respect to the y -axis?

- (1) $y = x$ (3) $y = \sin x$
(2) $y = x^2$ (4) $y = \tan x$

21 As angle x increases from 150° to 270° , the value of $\cos x$ will

- (1) increase from 0 to 1
- (2) increase from -1 to 0
- (3) decrease from 0 to -1
- (4) decrease from 1 to 0

22 The expression $\log \sqrt{xy}$ is equivalent to

- (1) $2 \log x \log y$
- (2) $2(\log x + \log y)$
- (3) $\frac{1}{2} \log x \log y$
- (4) $\frac{1}{2}(\log x + \log y)$

23 If, in $\triangle ABC$, $a = 5$, $b = 6$, and $c = 8$, then $\cos A$ is

- (1) $-\frac{1}{20}$
- (2) $\frac{11}{32}$
- (3) $\frac{25}{32}$
- (4) $\frac{53}{80}$

24 If $a = 4$, $b = 5$, and $m\angle A = 30$, the number of distinct triangles that may be constructed is

- (1) 1
- (2) 2
- (3) 3
- (4) 0

25 The maximum value of the function $y = 3 \sin 2x$ is

- (1) π
- (2) 2
- (3) 3
- (4) 2π

26 The expression $(\sin x - \cos x)^2$ is equivalent to

- (1) 1
- (2) $-\cos 2x$
- (3) $1 - \sin 2x$
- (4) $1 - \cos 2x$

27 The expression $\cos(90^\circ + \theta)$ equals

- (1) $\cos \theta$
- (2) $-\cos \theta$
- (3) $\sin \theta$
- (4) $-\sin \theta$

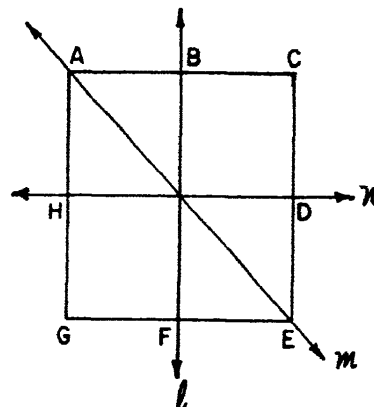
28 The graph of $y = ax^2 + bx + c$ is tangent to the x -axis. The roots of the equation $ax^2 + bx + c = 0$ are

- (1) real, rational, and unequal
- (2) real, rational, and equal
- (3) imaginary
- (4) real, irrational, and unequal

29 If $f(x) = \cos x + \tan \frac{x}{3}$, then $f(\pi)$ is

- (1) $\frac{\sqrt{3} + 3}{3}$
- (2) $\frac{\sqrt{3} - 3}{3}$
- (3) $\sqrt{3} + 1$
- (4) $\sqrt{3} - 1$

30 In the accompanying figure, ℓ , m , and n are symmetry lines. What is $r_\ell \circ r_m(\overline{CD})$?



- (1) \overline{AB}
- (2) \overline{CD}
- (3) \overline{EF}
- (4) \overline{GH}

31 The expression $\frac{5}{2 - \sqrt{3}}$ is equivalent to

- (1) $10 + 5\sqrt{3}$
- (2) $-2 - \sqrt{3}$
- (3) $-10 - 5\sqrt{3}$
- (4) $2 + \sqrt{3}$

32 If $\cos x = -\frac{4}{5}$ and $\tan x > 0$, then $\angle x$ terminates in Quadrant

- (1) I
- (2) II
- (3) III
- (4) IV

33 Which is the 7th term in the expansion of $(2x - y)^7$?

- (1) $7xy^6$
- (2) $-7xy^6$
- (3) $14xy^6$
- (4) $-14xy^6$

34 Which correctly compares the mean and median of the set of data shown in the accompanying table?

x_i measure	f_i frequency
60	2
75	4
80	1
90	3

- (1) The mean and median are equal.
- (2) The mean exceeds the median by 2.
- (3) The median exceeds the mean by 2.
- (4) The mean exceeds the median by 2.5.

35 On a standardized test, the mean is 61 and the standard deviation is 3.2. Which score can be expected to occur less than 3% of the time?

- (1) 50
- (2) 56
- (3) 62
- (4) 65

Answers to the following questions are to be written on paper provided by the school.

Part II

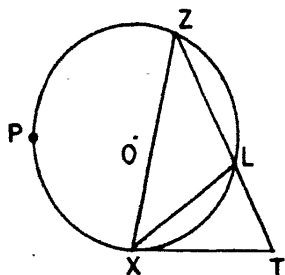
Answer four questions from this part. Show all work unless otherwise directed.

36 a On the same set of axes, sketch the graphs of $y = \frac{1}{2}\sin x$ and $y = \cos 2x$ as x varies from 0 to 2π radians. [8]

b From the graphs made in part a, determine the number of values of x between 0 and 2π radians which satisfy the equation $\frac{1}{2}\sin x = \cos 2x$. [2]

37 Find, to the nearest degree, all values of x in the interval $0^\circ \leq \theta \leq 360^\circ$ which satisfy the equation $3 \cos 2\theta + \sin \theta - 2 = 0$. [10]

38 Given: circle O , tangent \overline{TX} , secant \overline{TLZ} , chords \overline{ZX} and \overline{XL} , $m\widehat{XL}:m\widehat{LZ}:m\widehat{XPZ} = 2:2:5$.



- Find: a $m\widehat{XL}$ [2]
 b $m\angle Z$ [2]
 c $m\angle T$ [2]
 d $m\angle ZXT$ [2]
 e $m\angle XLT$ [2]

39 Given $A(8,5)$ and $B(6,1)$ and the transformations T , R , and S as described below:

$T: (x,y) \rightarrow (x+1,y-5)$

$R: (x,y) \rightarrow (y,x)$

$S: (x,y) \rightarrow (-x,y)$

a Graph \overline{AB} and its image $\overline{A'B'}$ after the transformation T . [3]

b Graph $\overline{A''B''}$, the image of \overline{AB} after the transformation R . [2]

c Graph $\overline{A'''B'''}$, the image of \overline{AB} after the transformation S . [2]

d Compare the slopes of the pairs of segments listed below and indicate whether these slopes are equal, reciprocals, additive inverses, or negative reciprocals.

(1) \overline{AB} and $\overline{A'B'}$ [1]

(2) \overline{AB} and $\overline{A''B''}$ [1]

(3) \overline{AB} and $\overline{A'''B'''}$ [1]

40 a Using logarithms, find $\sqrt[3]{0.692}$ to the nearest hundredth. [5]

b For all values of x for which the expressions are defined, prove the identity:

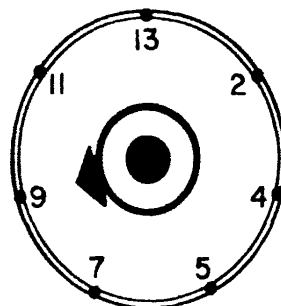
$$\frac{\tan \theta \csc^2 \theta}{1 + \tan^2 \theta} = \cot \theta \quad [5]$$

41 Two forces of 30 pounds and 40 pounds act upon a body, forming an acute angle with each other. The angle between the resultant and the 30-pound force is $35^\circ 10'$. Find, to the nearest ten minutes, the angle between the two given forces. [10]

42 Find, to the nearest tenth, the standard deviation for the following set of data. [10]

x_i measure	f_i frequency
60	1
75	4
80	3
90	2

43 The receivable channels on a TV are indicated on the channel selector shown. The probability of selecting each channel is the same.



a Find:

(1) $P(2)$ [1]

(2) $P(\text{even channel})$ [1]

(3) $P(\text{odd channel})$ [1]

b Find the probability of:

(1) choosing exactly two even channels on three random selections [3]

(2) choosing at least two odd channels on three random selections [4]

Formulas

Pythagorean and Quotient Identities

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 & \tan A &= \frac{\sin A}{\cos A} \\ \tan^2 A + 1 &= \sec^2 A & \cot A &= \frac{\cos A}{\sin A} \\ \cot^2 A + 1 &= \csc^2 A\end{aligned}$$

Functions of the Sum of Two Angles

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B}\end{aligned}$$

Functions of the Difference of Two Angles

$$\begin{aligned}\sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}\end{aligned}$$

Functions of the Double Angle

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ \cos 2A &= 2 \cos^2 A - 1 \\ \cos 2A &= 1 - 2 \sin^2 A \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}\end{aligned}$$

Functions of the Half Angle

$$\begin{aligned}\sin \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{2}} \\ \cos \frac{1}{2}A &= \pm \sqrt{\frac{1 + \cos A}{2}} \\ \tan \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}\end{aligned}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of Triangle

$$K = \frac{1}{2}ab \sin C$$

Standard Deviation

$$S.D. = \sqrt{\frac{1}{n} \sum_{i=1}^n (\bar{x} - x_i)^2}$$

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REGENTS HIGH SCHOOL EXAMINATION
SEQUENTIAL MATH — COURSE III
Tuesday, June 16, 1981 — 1:15 to 4:15 p.m., only

Part I Score:
Rater's Initials:

ANSWER SHEET

Pupil.....Teacher.....
School.....Grade:.....

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer 30 questions from this part.

- | | | | |
|---------|---------|---------|---------|
| 1..... | 11..... | 21..... | 31..... |
| 2..... | 12..... | 22..... | 32..... |
| 3..... | 13..... | 23..... | 33..... |
| 4..... | 14..... | 24..... | 34..... |
| 5..... | 15..... | 25..... | 35..... |
| 6..... | 16..... | 26..... | |
| 7..... | 17..... | 27..... | |
| 8..... | 18..... | 28..... | |
| 9..... | 19..... | 29..... | |
| 10..... | 20..... | 30..... | |

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Tuesday, June 16, 1981 — 1:15 to 4:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 19–35, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 8	(11) 1	(21) 2	(31) 1
(2) $18 + i$	(12) 25	(22) 4	(32) 3
(3) -1	(13) 70°	(23) 3	(33) 3
(4) 7	(14) 2.6332	(24) 2	(34) 2
(5) 1	(15) 24	(25) 3	(35) 1
(6) 6	(16) 150°	(26) 3	
(7) $-\sin 50^\circ$ or $-\cos 40^\circ$	(17) $\frac{\sqrt{5}}{5}$	(27) 4	
(8) 126°	(18) $\frac{13}{256}$	(28) 2	
(9) $(-3, -2)$	(19) 1	(29) 4	
(10) 2	(20) 2	(30) 3	

[OVER]

SEQUENTIAL MATH-COURSE III — *concluded*

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) b 4 [2]

(42) 8.1 [10]

(37) $30^\circ, 150^\circ, 199^\circ, 341^\circ$ [10]

(38) a 80 [2]

b 40 [2]

c 60 [2]

d 80 [2]

e 80 [2]

(43) a (1) $\frac{1}{7}$ [1]

(2) $\frac{2}{7}$ [1]

(3) $\frac{5}{7}$ [1]

(39) d (1) equal [1]

(2) reciprocals [1]

(3) additive inverses [1]

b (1) $\frac{60}{343}$ [3]

(2) $\frac{275}{343}$ [4]

(40) a 0.88 [5]

(41) $60^\circ 50'$ [10]