

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, August 14, 1996 — 8:30 to 11:30 a.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 & \tan A &= \frac{\sin A}{\cos A} \\ \tan^2 A + 1 &= \sec^2 A & \cot A &= \frac{\cos A}{\sin A} \\ \cot^2 A + 1 &= \csc^2 A\end{aligned}$$

Functions of the Sum of Two Angles

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B}\end{aligned}$$

Functions of the Difference of Two Angles

$$\begin{aligned}\sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}\end{aligned}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ \cos 2A &= 2 \cos^2 A - 1 \\ \cos 2A &= 1 - 2 \sin^2 A \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}\end{aligned}$$

Functions of the Half Angle

$$\begin{aligned}\sin \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{2}} \\ \cos \frac{1}{2}A &= \pm \sqrt{\frac{1 + \cos A}{2}} \\ \tan \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}\end{aligned}$$

Area of Triangle

$$K = \frac{1}{2}ab \sin C$$

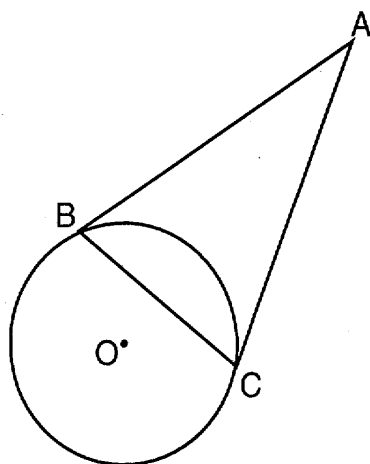
Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

- 1 In the accompanying diagram, \overline{AB} and \overline{AC} are tangents to circle O , and chord \overline{BC} is drawn. If $m\angle ABC = 72$, what is $m\angle A$?



- 2 Express the product in simplest form:

$$\frac{a^2 - 9}{a^2 - 3a} \cdot \frac{a^2 + a}{a + 3}$$

- 3 Express 225° in radian measure.

- 4 Solve for x : $\sqrt{2x - 2} - 2 = 0$

- 5 If the transformation $T_{(x,y)}$ maps point $A(1,-3)$ onto point $A'(-4,8)$, what is the value of x ?

- 6 A set of boys' heights is distributed normally with a mean of 58 inches and a standard deviation of 2 inches. Expressed in inches, between which two heights should 95% of the heights fall?

- 7 If $\sin A > 0$ and $\sec A < 0$, in which quadrant does the terminal side of $\angle A$ lie?

- 8 In which quadrant does the sum of $3 + 2i$ and $-4 - 5i$ lie?

- 9 Solve for x : $2^{x+2} = 4^{x-1}$

- 10 In $\triangle CAT$, $a = 4$, $c = 5$, and $\cos T = \frac{1}{8}$. What is the length of t ?

- 11 Evaluate: $\sum_{k=3}^7 (3k + 2)$

- 12 Express $\frac{5}{2 - i}$ in simplest $a + bi$ form.

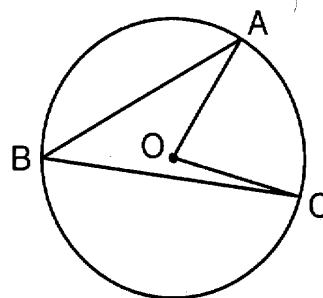
- 13 Solve for y : $y^{-\frac{1}{2}} = \frac{1}{3}$

- 14 In $\triangle ABC$, $a = 6$, $b = 7$, and $m\angle B = 30$. Find $\sin A$.

- 15 Solve for all values of x : $|3x - 2| = 16$

Directions (16–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

- 16 In the accompanying figure of circle O , $m\angle ABC = 38$.



What is $m\angle AOC$?

- (1) 19 (3) 76
(2) 38 (4) 152

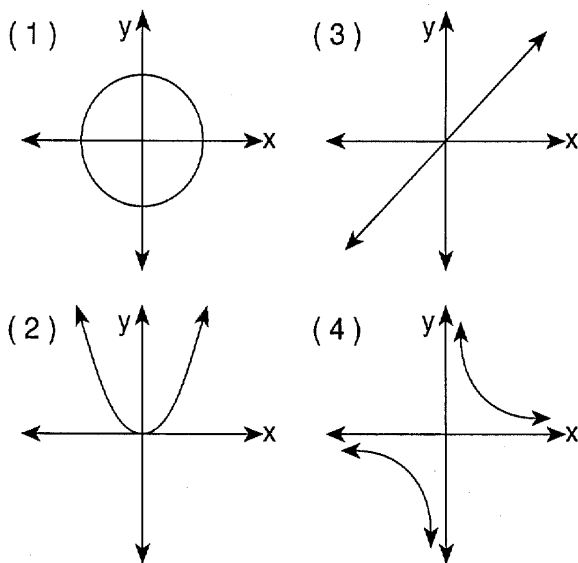
- 17 In a circle, diameter \overline{AB} is perpendicular to chord \overline{CD} at L . Which statement will always be true about this circle?
- (1) $CL = LD$ (3) $(CL) \times (LD) = AB$
 (2) $AL > LB$ (4) $BL > LA$

- 18 After which transformation of $\triangle ABC$ could the image $\triangle A'B'C'$ not have the same area?
- (1) translation (3) point reflection
 (2) rotation (4) dilation

- 19 The expression $\sin 50^\circ \cos 40^\circ + \cos 50^\circ \sin 40^\circ$ is equivalent to
- (1) $\sin 10^\circ$ (3) $\sin 90^\circ$
 (2) $\cos 10^\circ$ (4) $\cos 90^\circ$

- 20 If $\log_b x = y$, then $\log_b x^2$ is
- (1) $y + 2$ (3) $y - 2$
 (2) $2y$ (4) y

- 21 Which graph represents an inverse variation between all values of x and y ?



- 22 The graph of any function and the graph of its inverse are symmetric with respect to the
- (1) x -axis
 (2) y -axis
 (3) graph of the equation $y = -x$
 (4) graph of the equation $y = x$

- 23 If $m\angle B = 60$, $a = 6$, and $c = 10$, what is the area of $\triangle ABC$?
- (1) 15 (3) $15\sqrt{3}$
 (2) 30 (4) $30\sqrt{3}$

- 24 What is the period of the equation $y = -6 \sin 2x$?
- (1) $-\frac{2}{6}$ (3) 2π
 (2) -6π (4) π

- 25 The expression $\frac{1 + \cos 2x}{\sin 2x}$ is equivalent to
- (1) $\tan x$ (3) $-\sin x$
 (2) $\cot x$ (4) $-\cos x$

- 26 If $f(x) = 4 \cos 3x$, what is the value of $f(\frac{\pi}{4})$?
- (1) $-\sqrt{2}$ (3) 135
 (2) $-2\sqrt{2}$ (4) 4

- 27 For which value of θ is the fraction $\frac{6}{\cos \theta}$ undefined?
- (1) 0° (3) 60°
 (2) 30° (4) 90°

- 28 Which equation has roots of $3 + \sqrt{2}$ and $3 - \sqrt{2}$?
- (1) $x^2 + 6x + 7 = 0$ (3) $x^2 - 7x - 4 = 0$
 (2) $x^2 - 6x + 7 = 0$ (4) $x^2 - 7x + 6 = 0$

- 29 In basketball, Nicole makes 4 baskets for every 10 shots. If she takes 3 shots, what is the probability that *exactly* 2 of them will be baskets?
- (1) 0.288 (3) 0.600
 (2) 0.432 (4) 0.960

- 30 When two resistors are connected in a parallel circuit, the total resistance is $\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$. This

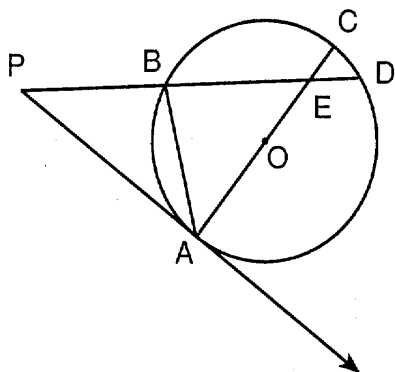
- complex fraction is equivalent to
- (1) $R_1 + R_2$ (3) $R_1 R_2$
 (2) $\frac{R_1 + R_2}{R_1 R_2}$ (4) $\frac{R_1 R_2}{R_1 + R_2}$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

- 36 In the accompanying diagram, \overrightarrow{PA} is tangent to circle O at point A , secant \overline{PBD} intersects diameter \overline{AC} at point E , chord \overline{AB} is drawn, $m\angle P = 40$, and $m\widehat{CD}:m\widehat{DA} = 1:8$.



Find:

- a $m\widehat{DA}$ [2]
 b $m\widehat{AB}$ [2]
 c $m\angle BEA$ [2]
 d $m\angle BAC$ [2]
 e $m\angle PBA$ [2]
- 37 a Solve for x : $x + \sqrt{2x - 1} = 8$ [5]
 b Solve for y : $\frac{y}{y - 1} = \frac{8}{y} + \frac{1}{y - 1}$ [5]
- 38 a Find, to the nearest degree, all values of x in the interval $0^\circ \leq x < 360^\circ$ that satisfy the equation $2 \sin^2 x = 1 + \sin x$. [5]
 b For all values of x for which the expression is defined, prove that the following is an identity:

$$\cot x = \frac{\sin 2x}{1 - \cos 2x} \quad [5]$$

- 39 a On graph paper, sketch the graph of the equation $y = \tan x$ in the interval $0 \leq x \leq 2\pi$. [4]
 b On the same set of axes, sketch the graph of the equation $y = 2 \sin x$ in the interval $0 \leq x \leq 2\pi$. [4]
 c Use the graphs sketched in parts a and b to determine one value of x in the interval $0 \leq x \leq 2\pi$ that satisfies the equation $\tan x = 2 \sin x$. [2]
- 40 Two forces act on a body at an angle of 100° . The forces are 30 pounds and 40 pounds.
 a Find the magnitude of the resultant force to the nearest tenth of a pound. [6]
 b Find the angle formed by the greater of the two forces and the resultant force to the nearest degree. [4]
- 41 a On the same set of axes, sketch and label the graphs of the equations $xy = 8$ and $y = \log_2 x$ in the interval $-6 \leq x \leq 6$. [8]
 b Using the graphs sketched in part a, find an integer value of x for which $\log_2 x > \frac{8}{x}$. [2]
- 42 a The probability of a biased coin coming up heads is $\frac{3}{4}$.
 (1) When the coin is flipped three times, what is the probability of at least two heads? [3]
 (2) When the coin is flipped four times, what is the probability of at most one head? [3]
 b Find the standard deviation, to the nearest hundredth, for the following test scores:
 100, 99, 99, 97, 96, 96, 95, 94, 93, 91 [4]

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REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Wednesday, August 14, 1996 — 8:30 to 11:30 a.m., only

Part I Score
Part II Score	<u>.....</u>
Total Score
Rater's Initials:

ANSWER SHEET

Pupil Sex: Male Female Grade

Teacher School

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

- | | | | |
|----------|----------|----------|----------|
| 1 | 11 | 21 | 31 |
| 2 | 12 | 22 | 32 |
| 3 | 13 | 23 | 33 |
| 4 | 14 | 24 | 34 |
| 5 | 15 | 25 | 35 |
| 6 | 16 | 26 | |
| 7 | 17 | 27 | |
| 8 | 18 | 28 | |
| 9 | 19 | 29 | |
| 10 | 20 | 30 | |

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS COURSE III

Wednesday, August 14, 1996 — 8:30 to 11:30 a.m., only

SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 16–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 36	(11) 85	(21) 4	(31) 3
(2) $a + 1$	(12) $2 + i$	(22) 4	(32) 1
(3) $\frac{5\pi}{4}$	(13) 9	(23) 3	(33) 2
(4) 3	(14) $\frac{3}{7}$	(24) 4	(34) 3
(5) -5	(15) $6, -\frac{14}{3}$	(25) 2	(35) 1
(6) 54 and 62	(16) 3	(26) 2	
(7) II	(17) 1	(27) 4	
(8) III	(18) 4	(28) 2	
(9) 4	(19) 3	(29) 1	
(10) 6	(20) 2	(30) 4	

[OVER]

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics*, 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

- | | | | |
|---|-----|-----------------------------------|-----|
| (36) <i>a</i> 160 | [2] | (40) <i>a</i> 45.6 | [6] |
| <i>b</i> 80 | [2] | <i>b</i> 40 | [4] |
| <i>c</i> 50 | [2] | | |
| <i>d</i> 50 | [2] | (41) <i>b</i> 5 or 6 | [2] |
| <i>e</i> 100 | [2] | | |
| | | (42) <i>a</i> (1) $\frac{54}{64}$ | [3] |
| (37) <i>a</i> 5 | [5] | (2) $\frac{13}{256}$ | [3] |
| <i>b</i> 8 | [5] | <i>b</i> 2.72 | [4] |
| | | | |
| (38) <i>a</i> 90, 210, 330 | [5] | | |
| | | | |
| (39) <i>c</i> 0 or π or 2π or $\frac{\pi}{3}$ or $\frac{5\pi}{3}$ | [2] | | |