

SOLID GEOMETRY

Monday, January 19, 1925 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including not more than four from group I.

Group I

Do not answer more than four questions from this group.

- 1 Prove that if two angles not in the same plane have their sides respectively parallel and lying in the same direction, they are equal. $[12\frac{1}{2}]$
- 2 Prove that the acute angle which a straight line makes with its own projection on a plane not perpendicular to the given line, is the least angle it makes with any line of that plane. $[12\frac{1}{2}]$
- 3 Prove that the volumes of two rectangular parallelepipeds are to each other as the products of their three dimensions. $[12\frac{1}{2}]$
- 4 Prove that every section of a convex cylinder made by a plane passing through an element is a parallelogram. $[12\frac{1}{2}]$
- 5 Prove that two triangles on the same sphere are either equal or symmetrical when the three angles of one are respectively equal to the three angles of the other. $[12\frac{1}{2}]$

Group II

Irrational results may be left in the form of π and radicals unless otherwise stated.

- 6 Find, if possible, the locus of a point in space equidistant from two parallel planes 6" apart and also (a) 4" from a point in one of the planes, (b) 3" from a point in one of the planes, (c) 2" from a point in one of the planes. $[4\frac{1}{2}, 4, 4]$

- 7 Find the volume of a frustum of a pyramid with altitude 10 if the upper and lower bases are rhombuses with edges 6 and 8 respectively and if one angle in each base is 60° . $[12\frac{1}{2}]$
- 8 Prove that if a line and a plane (not containing the line) are each perpendicular to a line, the first line and the plane are parallel. $[12\frac{1}{2}]$
- 9 An equilateral triangle with its inscribed circle is revolved about one of its altitudes as an axis. If one side of the triangle is 18, find the volume of the cone and the area of the surface of the sphere generated. $[6\frac{1}{2}, 6]$
- 10 The area of a zone is 30π and its altitude is 5. Find the area of a spheric triangle on the same sphere if the sides of its polar triangle are 70° , 80° and 90° . $[12\frac{1}{2}]$
- 11 A plane parallel to the base of a right circular cone cuts the altitude at a point $\frac{2}{3}$ of the distance from the vertex to the base. Find the ratio of (a) the volumes of the two solids formed $[6]$, (b) the lateral areas of the same two solids $[6\frac{1}{2}]$.