

SOLID GEOMETRY

Friday, January 25, 1929—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including not more than two from group I, one from group II, all the questions in group III and at least two questions from group IV.

Group I

Do not answer more than two questions from this group.

1 Prove that the locus of points equidistant from two given points is the plane perpendicular to the line joining them at its mid-point. [6, 6½]

2 Prove that a spheric angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides, produced if necessary. [12½]

3 a State a theorem about parallel planes used in proving that if a pyramid is cut by a plane parallel to its base the edges and altitude are divided proportionally. [2½]

b What theorem about the volume of a solid is used in proving that the volume of a triangular prism is equal to the product of its base and its altitude? [2½]

c Name the two surfaces whose areas are found by the formula $S = 2\pi rh$. [2½]

d Complete the following statement. [Do not prove.] [2½]
If two angles not in the same plane have their sides respectively parallel and extending in the same direction from their vertices, they are

e State the theorem about the face angles of a trihedral angle used in proving that the sum of the face angles of any polyhedral angle is less than four right angles. [2½]

Group II

Answer one question from this group.

4 A sphere is tangent to a plane. The radius of the sphere is 4". The locus of all points 1" from the surface of the sphere and 2" from the plane is to be determined.

a What kind of a geometric figure is the locus? [6½]

b Make a drawing showing the given sphere, the tangent plane and the locus. [6]

5 a Describe how to construct a plane perpendicular to a given plane and passing through two given points not in the given plane. [10]

b State the theorem that proves that the two planes are perpendicular. [2½]

Group III

Answer all the questions in this group.

Irrational results may be left in the form of π and radicals unless otherwise stated.

6 The altitude of a right circular cone is 8" and its slant height is 10"; find the radius of the sphere whose surface equals the total surface of the cone. [12½]

7 The altitude of a regular square pyramid is 9' and a diagonal of the base is 12'; what is the length of the edge of a cube that has the same volume as the pyramid? [12½]

8 The area of an isosceles spheric triangle is 6π square inches. It is on a sphere whose diameter is 12". Find the number of degrees in one base angle of the triangle if its vertex angle is 60° . [12½]

Group IV

Answer at least two questions from this group.

9 Prove that if two oblique lines, drawn from a point to a plane, meet the plane at equal distances from the foot of the perpendicular, they are equal, and conversely. [6½, 6]

10 Prove that a line parallel to each of two intersecting planes is parallel to their intersection. [12½]

11 Prove that the volumes of two cylinders of revolution whose lateral areas are equal, are to each other as their radii. [12½]

12 Copy and complete each of the following statements:

a The sum of the sides of a convex spheric polygon is less [2½]

b The volumes of two spheres are to each other as the [2½]

c A straight line perpendicular to each of two lines at their point of intersection is [2½]

d A plane angle of a dihedral angle is an angle formed by two lines [2½]

e $V = \frac{1}{3}\pi r^2 h$ is the formula for the . . . of a [2½]