

The University of the State of New York

265TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 23, 1936 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.1 Find the volume of a sphere whose radius is 3. [Answer may be left in terms of π .] Ans.....2 Write the formula for the lateral area S of a frustum of a right circular cone whose slant height is l and the radii of whose bases are r and r' . Ans.....3 One angle of a spheric triangle is 95° . How many degrees are there in the side opposite this angle in the polar triangle? Ans.....4 The angles of a spheric triangle are 80° , 50° and 110° . How many degrees are there in the spheric excess of the triangle? Ans.....5 If a line segment is $2\frac{1}{2}$ times as long as its projection on a plane, find the number of degrees in the acute angle which this line, extended if necessary, makes with the plane. Ans.....6 Find the lateral area of a regular square pyramid whose lateral edge is 5 and whose base edge is 6. Ans.....7 What is the length of a diagonal of a rectangular parallelepiped whose dimensions are $6 \times 3 \times 2$? Ans.....8 An element e of a right circular cone meets the base of the cone at an angle of A degrees. Express the height of the cone in terms of e and A . Ans.....9 The base of a prism is a rhombus whose diagonals are 12 feet and 16 feet. If the height of the prism is 10 feet, what is the volume of the prism expressed in cubic feet? Ans.....10 A lune whose angle is 90° has an area of 81π square inches. Find the radius of the sphere on which this lune is drawn. Ans.....11 A cylinder and a cone have equal bases and equal volumes. If the altitude of the cylinder is h , what is the altitude of the cone? Ans.....12 How many faces has a regular convex polyhedron that has 8 vertices and 12 edges? Ans.....13 What fractional part of the earth's surface lies north of latitude 30° north? [Assume the earth to be a sphere.] Ans.....14 The locus of points in space equidistant from two given intersecting lines is (a) two intersecting lines, (b) a plane, (c) two intersecting planes or (d) two parallel planes. Which is correct, (a), (b), (c) or (d)? Ans.....Directions (questions 15-19) — Indicate whether each of the statements 15-19 is *always true*, *sometimes true* or *never true* by writing the word *always*, *sometimes* or *never* on the dotted line at the right.15 Two straight lines, each parallel to a given plane, are parallel to each other. Ans.....

16 Through a given line oblique to a given plane, one and only one plane can be passed perpendicular to the given plane.

Ans.

17 The locus of points equidistant from two given points and at a given distance from the line passing through the given points is a circle.

Ans.

18 An exterior angle of a spheric triangle is equal to the sum of the two opposite interior angles of the triangle.

Ans.

19 The three face angles of a trihedral angle may be in the ratio 1 : 2 : 3.

Ans.

20 The volume of a pyramid is 81 cubic inches. A plane passing through the pyramid parallel to the base cuts off a pyramid whose volume is 3 cubic inches. What is the ratio of the base of the larger pyramid to the base of the smaller pyramid?

Ans.

See instructions for groups II and III on page 1.

Group II

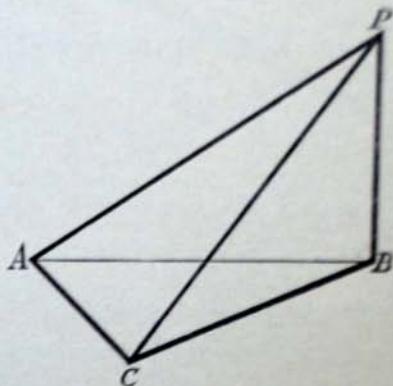
Answer three questions from this group.

21 Prove that if two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]

22 Prove that a spheric angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides, produced if necessary. [10]

23 Prove that the lateral areas of any two similar cones of revolution are to each other as the squares of their radii. [10]

24 ABC is a right triangle with AB the hypotenuse and P a point outside the plane of the triangle. Lines PC and PB are drawn. If PC is perpendicular to AC and plane PAB is perpendicular to plane ABC , prove that PB is perpendicular to the plane ABC . [10]



*25 Discuss fully two of the following loci:

- The locus of points at a given distance d from a given circle whose radius is r , where r is greater than d . [5]
- The locus of points equidistant from a given plane and a given line perpendicular to the plane at a given point. [5]
- The locus of points equidistant from a given plane and a given point outside the plane. [5]

Group III

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Use $\pi = 3\frac{1}{7}$ unless otherwise stated.

26 The sides of a spheric triangle are 85° , 70° and 105° . Find in square inches the area of the polar triangle, the radius of the sphere being 12 inches. [Answer may be left in terms of π .] [10]

27 A rectangle whose base is b and whose height is h is revolved, first about b as an axis and then about h as an axis. Show that the ratio between the volumes of the two solids of revolution is the same as the ratio between the total surfaces of the two solids. [10]

28 The slant height of a frustum of a regular square pyramid makes with the larger base an angle of 65° . The base edges are 4 inches and 3 inches. Find the volume correct to the nearest cubic inch. [10] [Formula: $V = \frac{h}{3}(b + b' + \sqrt{bb'})$]

* This question is based on one of the optional topics in the syllabus and may be used in either group II or group III.