

The University of the State of New York
268TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 21, 1937 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-4) — Indicate the correct answer to each of the following questions by writing on the dotted line at the right the letter *a*, *b* or *c*.

1 The surface formed by all the straight lines which intersect a given straight line at a given point and which make a given acute angle with that line is (a) plane, (b) cylindrical or (c) conical.

Ans.....

2 If the corresponding edges of two similar solids are in the ratio 3:4, their volumes are in the ratio (a) 3:4, (b) 9:16 or (c) 27:64

Ans.....

3 The area of a lune is to the area of the sphere on which it is drawn as the angle of the lune is to (a) 180° , (b) 360° or (c) 720° .

Ans.....

4 The spheric excess of a spheric triangle (a) may be greater than 360° , (b) must be less than 180° or (c) must be less than 360° .

Ans.....

Directions (questions 5-9) — Indicate whether each of the following statements is *always* true, *sometimes* true or *never* true by writing on the dotted line at the right the word *always*, *sometimes* or *never*.

5 If a line is parallel to each face of a dihedral angle, it is parallel to the edge of the dihedral angle.

Ans.....

6 Through any given point one and only one line can be drawn perpendicular to each of two parallel planes.

Ans.....

7 Two planes perpendicular to the same plane are parallel.

Ans.....

8 If two lines are perpendicular, a plane which contains one and only one of these lines is perpendicular to the other line.

Ans.....

9 If two unlimited lines do not lie in the same plane, a plane can be drawn which is perpendicular to both of these lines.

Ans.....

Directions (questions 10-20) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

10 The total area of a regular tetrahedron whose edge is e is given by the formula $A = \dots$

Ans.....

11 The sum of the face angles of any convex polyhedral angle is less than \dots right angles.

Ans.....

12 The diagonal d of a rectangular parallelepiped whose edges are a , b and c is given by the formula $d = \dots$

Ans.....

13 If the area of a right section of an oblique prism is 12 square inches and the lateral edge is 15 inches, the volume of the prism is \dots cubic inches.

Ans.....

14 A sphere is circumscribed about a cube whose diagonal is 10 inches. The volume of the sphere is \dots cubic inches. [Answer may be left in terms of π .]

Ans.....

15 The area of a sphere is 64π square inches. The height of a zone drawn on this sphere is one inch. The area of the zone is ... square inches. [Answer may be left in terms of π .]

Ans.....

16 The area of a spheric triangle whose angles are 100° , 80° and 60° , drawn on a sphere whose radius is 2 inches, is ... square inches. [Answer may be left in terms of π .]

Ans.....

17 $ABCD$ is a trapezoid in which the lower base AD is 4 inches, the upper base BC is 3 inches, the angle A is 45° and the angle D is 90° . The lateral area of the solid formed by revolving the trapezoid about CD as an axis through an angle of 360° , is ... square inches. [Answer may be left in terms of π and radicals.]

Ans.....

18 The area of the base of a cone is b square inches. A plane passing through the mid-point of an element of the cone, parallel to the base, forms a section of the cone whose area is ... square inches.

Ans.....

19 If the side of the base and the height of a regular square pyramid are each multiplied by n , the volume of the pyramid is multiplied by ...

Ans.....

20 If the lateral area of a regular square pyramid is twice the area of the base, each lateral face of the pyramid is inclined to the base at an angle of ... degrees.

Ans.....

See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

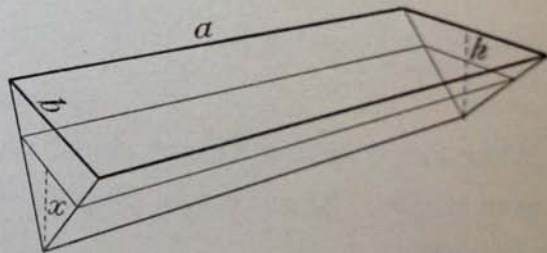
- 21 Prove that the sum of any two face angles of a trihedral angle is greater than the third face angle. [10]
- 22 Prove that in two polar triangles, each angle of one has the same measure as the supplement of the side lying opposite to it in the other. [10]
- 23 Given line m and two parallel planes, P and Q
- a If m is perpendicular to the planes, describe completely the locus of points that are equidistant from P and Q and also at a given distance d from m . [5]
- b What change takes place in the locus found in answer to a if the line m becomes oblique to P ? [2]
- c Describe a position of m in which there would be no solution to the problem. [3]
- 24 Three planes, P , Q and R , intersect in a line. From a point M , external to the three planes, lines a , b , and c are drawn perpendicular to P , Q and R respectively. Prove that a , b and c lie in the same plane. [10]
- *25 Using Cavalieri's Theorem, prove that the volume of any prism is equal to the product of its base and its altitude. [10]

Group III

Answer two questions from this group.

26 A well is to have the form of a right circular cylinder 4 feet in diameter and 30 feet deep. A contractor estimates that it will cost 25¢ per cubic foot to dig the well and 15¢ per square foot to lay a stone surface on the side and bottom. Find the cost of construction. [Use $\pi = 3.14$] [10]

27 A watering trough has the form of a right triangular prism as shown in the figure. The rectangle at the top of the trough is a feet long and b feet wide and the depth of the trough is h feet.



- a Show that the number of cubic feet of water, V , in the trough, when it is filled to a depth of x feet, is given by the formula $V = \frac{abx^2}{2h}$ [8]
- b How many gallons of water are there in the trough if $a = 12$ feet, $b = 2$ feet, $h = 15$ inches and $x = 10$ inches? [$7\frac{1}{2}$ gallons = 1 cubic foot] [2]
- 28 The slant height s of a cone of revolution makes with its projection on the base an angle x .
- a Express the volume V of the cone as a function of s and x . [5]
- b Find V correct to the nearest tenth of a cubic inch if $s = 1.8$ inches and $x = 53^\circ$ [Use $\pi = 3.14$] [5]
- * This question is based on one of the optional topics in the syllabus.