

The University of the State of New York

302D HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 29, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that a spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides, produced if necessary. [10]

22 Prove that if a pyramid is cut by a plane parallel to its base,

a The edges and altitude are divided proportionally [4]

b The section is a polygon similar to the base [6]

23 Prove that a diagonal of a parallelepiped bisects any line passing through its mid-point and terminating in any two opposite faces. [10]

24 Given planes P and Q intersecting in line MN , describe the locus of all points

a Equidistant from planes P and Q [2]

b At a given distance r from MN [2]

c At a given distance s from plane P [2]

d That satisfy both conditions given in a and b [2]

e That satisfy both conditions given in a and c [2]

Part III

Answer three questions from part III.

25 A spherical triangle whose sides are 85° , 90° and 115° is drawn on a sphere whose radius is 6". Find in square inches the area of the polar triangle. [10]

26 The volume of a sphere is equal to that of a right circular cylinder whose diameter d is equal to its altitude.

a Express radius r of the sphere as a function of d . [6]

b Find r to the nearest tenth if d equals 12. [4]

[1]

[OVER]

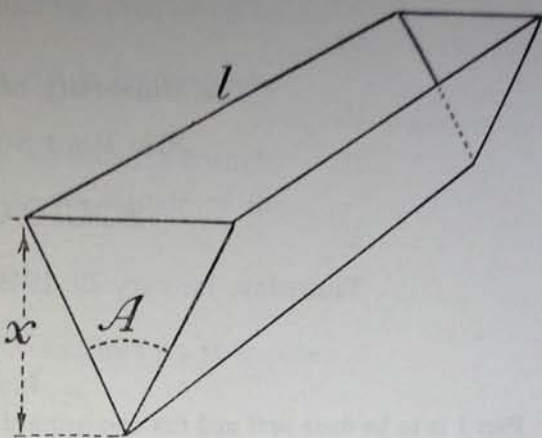
27 An open trough has the form of a right triangular prism as shown in the figure. The length is l and the ends are isosceles triangles with vertex angle A .

a Show that the volume of the trough for any depth x is $V = x^2 l \tan \frac{A}{2}$ [4]

b Show that the outside area of the trough for any depth x is

$$K = 2x^2 \tan \frac{A}{2} + \frac{2xl}{\cos \frac{A}{2}} \quad [4]$$

c Find to the nearest square foot the outside area of this trough if $x = 1$ foot, $l = 6$ feet and $A = 90^\circ$ [2]



28 An open cistern has the form of a frustum of a cone of revolution whose depth is 6 feet, top diameter 10 feet and bottom diameter 5 feet.

a At 2 cents per square foot, find to the nearest dollar the cost of painting the inside of the cistern. [6]

b Find to the nearest 10 gallons the capacity of the cistern. $[V = \frac{\pi h}{3} (r_1^2 + r_2^2 + r_1 r_2)]$

$$[1 \text{ gal.} = 231 \text{ cu. in.}] \quad [4]$$

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-6) — Write on the dotted line at the right of each question the expression which, when inserted in the blank, will make the statement true.

- 1 Through a given point outside a given plane one and only one line can be drawn ... to the plane. 1.....
- 2 The volumes of two prisms having equal bases are to each other as their 2.....
- 3 If the plane angle of a dihedral angle is 60° , a point 10 inches from each face of the dihedral angle is ... inches from the edge. 3.....
- 4 If two face angles of a trihedral angle are 90° and 100° , the third face angle must be greater than ... degrees and less than ... degrees and may have any value between these limits. 4.....
- 5 The sum of the angles of a spherical triangle must be less than ... degrees. 5.....
- 6 A lune whose angle is 30° contains ... spherical degrees. 6.....

Directions (questions 7-13) — In each of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

- 7 A line segment oblique to a plane is greater than its projection on the plane. 7.....
- 8 If two lines are parallel to the same plane, they are parallel to each other. 8.....
- 9 If a plane is parallel to one line and perpendicular to another line, the two lines lie in the same plane. 9.....
- 10 Two planes are perpendicular if one contains a line which is perpendicular to the other. 10.....
- 11 Any line which meets a plane obliquely will meet some line in the plane at right angles. 11.....
- 12 PP' and RR' are two perpendicular diameters of sphere O ; then PP' is perpendicular to any great circle containing RR' . 12.....
- 13 All great circles of a sphere bisect each other. 13.....
- 14 Express the total area of a right circular cylinder as a function of its radius r and its altitude h . 14.....
- 15 A solid spherical ball 4 inches in diameter weighs 10 pounds. Find the weight of a ball of the same material 8 inches in diameter. 15.....
- 16 The radius of a sphere is 6 inches. Find in square inches the area on the sphere included between two parallel planes 3 inches apart. [Answer may be left in terms of π .] 16.....
- 17 Find the diagonal of a rectangular solid whose dimensions are 2, 6 and 9. 17.....
- 18 A right triangle whose legs are 5 and 12 is revolved through 360° about the longer leg as an axis. Find the lateral area of the solid formed. [Answer may be left in terms of π .] 18.....
- 19 Find the total area of a regular tetrahedron whose edge is 3. [Answer may be left in radical form.] 19.....
- 20 The volume of a right circular cone is 48π and its altitude is 9. Find the radius of the base. 20.....