SOLID GEOMETRY

Thursday, January 27, 1949 — 9.15 a. m. to 12.15 p. m., only

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-5) — In *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false.*

1. If two planes are perpendicular to each other, a line perpendicular to one of them from any point in the other lies in the other.   1...............

2. If a line is parallel to a plane, all planes passing through this line are parallel to the plane.   2...............

3. If a line is parallel to both faces of a dihedral angle, it is parallel to the edge of the angle.   3...............

4. If two angles not in the same plane have their sides respectively parallel, the angles are equal.   4...............

5. The locus of points equally distant from two given parallel planes and at a given distance from a point midway between the two given planes is a circle.   5...............

Directions (questions 6-10) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b,* or *c.*

6. If a polyhedron is regular,   (*a*) it may have 10 faces (*b*) it may have 12 edges and 6 vertices   (*c*) its faces may be regular hexagons   6........

7. In any parallelepiped,   (*a*) the diagonals are equal (*b*) the plane passed through any two diagonally opposite edges divides the parallelepiped into two congruent prisms   (*c*) its volume is equal to the product of its lateral edge and the area of its right section   7........

8. If the bases of any two similar cones of revolution are in the ratio 1 : 4, (*a*) their volumes are in the ratio 1 : 16 (*b*) their total areas are in the ratio 1 : 4   (*c*) their altitudes are in the ratio 1 : 4   8........

9. If a pyramid is cut by a plane parallel to its base and bisecting its altitude, the volume of the pyramid cut off by the plane is to the volume of the remaining frustum as   (*a*) 1 : 4   (*b*) 1 : 8   (*c*) 1 : 7   9........

10. A plane is *determined* if it passes through a given point and (*a*) is perpendicular to a given plane   (*b*) is parallel to both of two given skew lines   (*c*) is parallel to a given line   10........

Directions (questions 11-17) — Write the answer to *each* question on the line at the right.

11. Find the lateral area of a regular pentagonal pyramid whose base edge is 4 and whose slant height is 10.   11................

12. Express the lateral area of a frustum of a right circular cone in terms of its slant height l and the radii r and r' of its bases.   12................

13. A cube is inscribed in a sphere whose diameter is d. Express the edge of the cube in terms of d. [Answer may be left in radical form.]   13................

14. Find the volume of a circular cone whose radius is 3 and whose altitude is 9. [Answer may be left in terms of π.]   14................

15. Find the volume of a sphere whose radius is 3. [Answer may be left in terms of π.]   15................

16. The radius of a small circle of a sphere is 3 inches and the plane of the circle is 4 inches from the center of the sphere. Find the area of the sphere. [Answer may be left in terms of π.]   16................

17. A zone and a lune drawn on the same sphere are equal. If the altitude of the zone is 1 and the angle of the lune is 45°, find the radius of the sphere.   17................

Directions (questions 18-20) — Answer *yes* or *no* on the line at the right.

18. Must two spherical triangles drawn on the same sphere be either congruent or symmetric if *any* three parts of one are equal to the three corresponding parts of the other?   18........

19. If two face angles of a trihedral angle are 80° and 120°, can the third face angle be 160°?   19........

20. If the section of a cone made by a plane parallel to the base is a circle, is the cone necessarily a circular cone?   20........

Part II

Answer two questions from part II.

21. *a.* Prove that any point in the plane which is perpendicular to a given line segment at its mid-point is equidistant from the end points of the segment. [8]

    b. State the converse of the theorem given in *a.* [2]

22. Prove that the sum of the dihedral angles of a trihedral angle is greater than 180° and less than 540°. [Suggestion: Draw a sphere whose center is the vertex of the trihedral angle.] [10]

23. Prove that two lines perpendicular to the same plane are parallel. [10]

*24. The accompanying figure represents a wedge. $ABCD$ is a rectangle and EF is a line segment parallel to AB. Using the prismatoid formula, $V = \dfrac{h}{6}\ (B + B' + 4M)$, find to the *nearest tenth of a cubic foot* the volume of the wedge if $AB = 3$ feet, $AD = 10$ inches, $EF = 2\frac{1}{2}$ feet, and the distance from EF to the plane of the rectangle is 15 inches. [10]
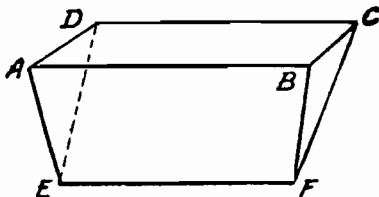
*This question is based on one of the optional topics in the syllabus and may be used in either part II or part III.

Part III

Answer three questions from part III.

25. A concrete pillar has the form of a frustum of a right circular cone surmounted by a right circular cylinder whose base is the upper base of the frustum. The upper and lower bases of the frustum are 2 feet and 3 feet respectively in diameter, the height of the frustum is $1\frac{1}{2}$ feet and the height of the cylinder is 12 feet. Find to the *nearest cubic foot* the volume of the pillar. [10]

The formula for the volume of a frustum is

$$V = \frac{\pi h}{3}\ (r_1{}^2 + r_2{}^2 + r_1\,r_2) \quad [\text{Use } \pi = 3.14]$$

26. The angles of a spherical triangle are $80°$, $85°$ and $105°$. If the radius of the sphere on which the triangle is drawn is represented by r, express in terms of r

    a. The area of the triangle [4]

    b. The perimeter of the polar triangle [6]

27. A solid wooden pedestal has the form of a regular hexagonal prism whose base edge is 8 inches and whose height is 15 feet. Find to the *nearest pound* the weight of the pedestal if a rectangular block of the same material, 2 inches by 10 inches by 3 feet, weighs 20 pounds. [Use $\sqrt{3} = 1.73$] [10]

28. The altitude of a regular square pyramid is h and a lateral edge of the pyramid makes with its projection on the base an angle x.

    a. Express the volume V of the pyramid in terms of h and x. [6]

    b. Find V to the *nearest cubic inch* if $h = 4.3$ inches and $x = 42°$. [4]