

# SOLID GEOMETRY

Wednesday, January 26, 1955—9.15 a.m. to 12.15 p.m., only

## Part I

Answer all questions in this part. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed.

1. The altitude of a right prism is 10 and its base is a triangle whose sides are 3, 4 and 6. Find the lateral area of the prism. 1.....

2. A base edge of a regular triangular pyramid is  $e$  and the slant height is  $s$ . Express the lateral area of this pyramid in terms of  $e$  and  $s$ . 2.....

3. A cylinder and a cone have equal bases and equal altitudes. If the volume of the cylinder is 12, find the volume of the cone. 3.....

4. The dimensions of the base of a rectangular parallelepiped are 6 and 9 and a diagonal of the parallelepiped is 11. Find the altitude of the parallelepiped. 4.....

5. The area of the base of a pyramid is 16. Find the area of the section which is parallel to the base and bisects the altitude. 5.....

6. A solid spherical ball 4 inches in diameter weighs 8 pounds. Find the weight of a solid spherical ball of the same material which is 6 inches in diameter. 6.....

7. Find the volume of a sphere whose radius is 6. [Answer may be left in terms of  $\pi$ .] 7.....

8. The sum of the angles of a spherical triangle is  $216^\circ$ . Find the number of square inches in the area of the triangle if the radius of the sphere is 5 inches. [Answer may be left in terms of  $\pi$ .] 8.....

9. The area of a lune is 60 spherical degrees. What fractional part of the area of the sphere is the area of this lune? 9.....

*Directions (10-15):* Indicate the correct completion for each of the following by writing on the line at the right the letter  $a$ ,  $b$  or  $c$ .

10. A regular polyhedron may have (a) 4 vertices and 8 edges (b) 6 vertices and 12 edges (c) 6 vertices and 8 edges 10.....

11. A spherical quadrilateral lies on a sphere whose radius is 1 inch. The perimeter of the quadrilateral in inches must be (a) less than  $2\pi$  (b) equal to  $2\pi$  (c) more than  $2\pi$  11.....

12. Line segment  $AB$  makes an acute angle  $x$  with plane  $Q$ . The length of the projection of  $AB$  on  $Q$  is (a)  $AB \sin x$  (b)  $AB \cos x$  (c)  $AB \tan x$  12.....

13. Two face angles of a trihedral angle are  $80^\circ$  and  $120^\circ$ . The third angle may be (a)  $40^\circ$  (b)  $80^\circ$  (c)  $160^\circ$  13.....

14. If the radius of the base of a right circular cone is divided by 2 and its slant height is multiplied by 2, the lateral area of the cone (a) is decreased (b) remains the same (c) is increased 14.....
15. The locus of points at a given distance  $d$  from a plane  $Q$  and at a given distance  $r$  from a point on the plane  $Q$  may be (a) one point (b) one circle (c) two circles 15.....
- Directions (16-20): For each of the following, if the statement is always true, write the word true on the line at the right; if it is not always true, write the word false.*
16. Only one great circle can be drawn through two points on a sphere. 16.....
17. In any circular cone, the axis passes through the center of every section made by a plane parallel to the base of the cone. 17.....
18. If a line in one plane is perpendicular to a line in another plane, the planes are perpendicular to each other. 18.....
19. The projections of two skew lines on a plane are parallel. 19.....
20. If two planes are perpendicular to a line, their intersections with any plane through the line are parallel. 20.....

## Part II

*Answer two questions from this part.*

21. Prove: If a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [10]
22. Given an unlimited straight line  $m$  and line segment  $PQ$  8 inches long.
- A. Describe fully the locus of points
- (1) 5 inches from  $m$  [3]
  - (2) equally distant from  $P$  and  $Q$  [3]
- B. Indicate the correct completion for each of the following by writing the letter  $a$ ,  $b$  or  $c$  after the proper number on your answer paper:
- (1) If  $P$  is on  $m$  and  $PQ$  is perpendicular to  $m$ , the locus of points satisfying both conditions given in A consists of (a) one line (b) 2 parallel lines (c) a circle [2]
  - (2) If both  $P$  and  $Q$  lie on  $m$ , the locus of points satisfying both conditions given in A consists of (a) one line (b) 2 parallel lines (c) a circle [2]
23. Prove: The sum of the angles of a spherical triangle is greater than  $180^\circ$  and less than  $540^\circ$ . [10]
24.  $ABCD-RSTW$  is a parallelepiped whose lateral edges are  $AR$ ,  $BS$ ,  $CT$ , and  $DW$ . The plane through  $RW$  and  $BC$  intersects the plane through  $ST$  and  $AD$  in  $PQ$ . Prove that  $PQ$  is parallel to plane  $AC$ . [10]

## Part III

Answer three questions from this part. Show all work.

25. A plane divides a sphere into two zones whose areas are in the ratio 1:3. If the altitude of the larger zone is 9, find
- the altitude of the smaller zone [3]
  - the area of the smaller zone [Answer may be left in terms of  $\pi$ .] [4]
  - the angle of a lune on the sphere if the area of this lune is equal to the area of the smaller zone [3]

26. The cross section of a ditch 14 feet long has the form of a trapezoid whose bases are 4 feet and 6 feet and whose altitude is 8 feet. A cylindrical pipe whose diameter is 3 feet is laid horizontally the entire length of the ditch. Find, to the nearest cubic yard, the amount of gravel that will be needed to fill the remaining space in the ditch. [10]

27. A container is to be made in the shape of a frustum of a cone of revolution. The radius of the upper base is to be 6 inches and of the lower base 3 inches, and the container is to have a volume of 264 cubic inches.

- Find the depth of the container.

[Use  $V = 1/3 \pi h(r_1^2 + r_2^2 + r_1 r_2)$  and  $\pi = 22/7$ .] [4]

- Find to the nearest square inch the amount of material necessary to make the container if it is open at the top and no allowance is made for waste. [6]

28. In the accompanying figure the angles at  $P$  and  $S$  are right angles,  $QR$  is a quadrant of a circle,  $PS = a$  and  $PQ = b$ . Show that the total area  $T$  of the solid formed by revolving the figure through  $360^\circ$  about  $PQ$  as an axis is given by the formula  $T = \pi a(5a + 2b)$ . [10]

