The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Thursday, January 25, 1962 — 1:15 to 4:15 p.m., only

Name of pupil.....Name of school.....

Name and author of textbook used.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 How many inches from the center of a sphere of radius 10 is the plane of a small circle of radius 6? 1.....

2 Find the radius of a sphere whose area is 196π. 2.....

3 A line segment 10 inches long makes an angle of 60° with its projection in a plane. Find the length of the projection in inches. 3.....

4 In a certain right circular cylinder, the altitude is twice the radius r. Express the lateral area in terms of r. 4.....

5 The angles of a spherical triangle on a sphere of radius 18 inches are 120°, 90°, 70°. Find the area of the triangle in square inches. 5.....

6 A sphere of radius 5 inches is cut into 2 zones by a plane 4 inches from the center of the sphere. Find the number of square inches in the area of the smaller zone. 6.....

7 Find the volume of a prism if the altitude is 10 and the base is an equilateral triangle whose side is 8. 7.....

8 An element of a right circular cone is 12 and makes an angle of 45° with the axis of the cone. Find the altitude of the cone. 8.....

9 A point P within a dihedral angle of 62° is located 4 inches from each of the planes of the angle. Find to the *nearest inch* the distance from P to the edge of the dihedral angle. 9.....

10 Find the volume of a pyramid of altitude $2\sqrt{3}$, whose base is a triangle with sides 2, 1, $\sqrt{3}$. 10.....

[OVER]

11 Find the lateral area of a regular pyramid that has a square base of side 3 and a slant height equal to the diagonal of the base. 11.....

12 The base of a right prism is a rectangle whose sides are 3 and 5. If the altitude of the prism is $\sqrt{15}$, find the length of a diagonal of the prism. 12.....

13 A lune with an area of 64π square inches is drawn on a sphere of radius 8 inches. Find the number of degrees in the angle of the lune. 13.....

14 A plane divides a cube of edge 6 into two congruent triangular prisms. Find the area of the rectangular face common to both prisms. 14.....

15 Find the area of a regular octahedron whose edge is 4. 15.....

16 The ratio of the lateral areas of two similar cylinders of revolution is 1:16. Find the ratio of the volume of the smaller cylinder to the volume of the larger. 16.....

17 A pyramid has an altitude of 8 inches. If the area of the base is 16 square inches, find the area in square inches of the section made by a plane passed parallel to and 3 inches from the base. 17.....

18 The volume of a hemisphere is 144π. Find the radius. 18.....

19 An edge of a regular tetrahedron is 8. Find the lateral area of the frustum which is formed by a plane intersecting three lateral edges of the tetrahedron at their midpoints. 19.....

Directions (20–25): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

20 If the radius of a right circular cone is divided by 2 and the altitude is multiplied by 2, the volume

(1) remains the same
(2) is multiplied by 2
(3) is divided by 2
(4) is divided by 4

20.....

21 Points A and B are 6 inches apart. Point P moves so that its distances from A and B are each equal to 5 inches. What is the locus of P?

(1) a sphere of radius 5 inches
(2) a circle of radius 4 inches
(3) two circles of radius 4 inches
(4) two spheres of radius 3 inches

21.....

22 A rectangle with sides a and b is rotated 360° about side a as an axis. The volume of the solid thus generated is

(1) $\pi a^2 b$
(2) $\pi a b^2$
(3) $2\pi ab$
(4) $2\pi b(a+b)$

22.....

23 Two face angles of a trihedral angle are 100° and 130°. The third face angle may be

(1) 30°
(2) 140°
(3) 130°
(4) 90°

23.....

24 Line n is oblique to plane P. The number of planes which can be passed through n, perpendicular to P, is

(1) one
(2) two
(3) infinite
(4) zero

24.....

25 The Northern Hemisphere is divided into two zones by the 30° north parallel of latitude. The ratio of the areas of the two zones is

(1) 1:1
(2) 1:2
(3) 1:3
(4) $\sqrt{3}:2$

25.....

Directions (26–30): If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

26 If lines drawn from a point in a perpendicular to a plane meet the plane at equal distances from the foot of the perpendicular, they are ... equal. 26.....

27 The diagonals of a skew quadrilateral ... intersect. 27.....

28 Two planes are ... parallel if they are parallel to the same line. 28.....

29 If a line is parallel to one plane and perpendicular to another plane, the two planes are ... perpendicular to each other. 29.....

30 If three planes are mutually perpendicular, their lines of intersection are ... concurrent. 30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b*: [10]

a If two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other.

OR

b In two polar triangles each angle of one has the same measure as the supplement of the side lying opposite it in the other.

32 Answer *either* *a* or *b*: [10]

a Plane M and line a outside plane M are both perpendicular to line l. Prove line a is parallel to plane M.

OR

b Using the theorem, " Two spherical triangles are congruent or symmetric if the three sides of one are respectively equal to the three sides of the other," prove that two spherical triangles are congruent or symmetric if the three angles of one are respectively equal to the three angles of the other.

33 Given line n and plane P *not* containing n,

a (1) describe fully the locus of points d units from n [2]
(2) describe fully the locus of points s units from P [2]

b name the locus of points satisfying both conditions in part *a* if n is parallel to P and 4 units from P and
(1) $d = 3, s = 5$ [2]
(2) $d = 3, s = 1$ [2]

c If n is perpendicular to P, name the locus of points satisfying both conditions in part *a*. [2]

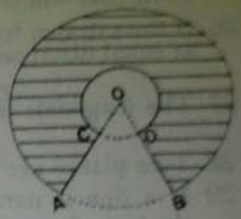
34 A lateral edge of a regular square pyramid makes an angle of θ degrees with its projection on the base. The side of the base is s and the height of the pyramid is h. A plane is passed between the base and the vertex p units from the base and parallel to it so as to cut off a smaller pyramid.

a Express s in terms of h and $\tan \theta$. [4]

b Show that a formula for the volume of the smaller pyramid is $V = \frac{2(h-p)^3}{3 \tan^2 \theta}$. [6]

35 The flat pattern in the accompanying diagram is used to make a lampshade which is the lateral surface of the frustum of a right circular cone, as follows:

The two concentric circles have radii of 12 inches and 4 inches, and $\angle AOB = 60°$. The minor sector AOB and the remainder of the interior of the smaller circle are removed and discarded. AC is fastened to BD without waste.



a Find the radius of each of the *two* bases of the lampshade. [4]

b Find the area of the outer surface of the lampshade. [Leave answer in terms of π.] [3]

c Find the altitude of the lampshade. [Leave answer in radical form.] [3]

*36 *a* Describe fully the surface represented by the equation $x^2 + y^2 + z^2 = 25$. [2]

b Describe fully the surface $x = 3$. [2]

c Name the locus which is the intersection of the surfaces $x^2 + y^2 + z^2 = 25$ and $x = 3$. [2]

d Find the coordinates of the points of intersection of the locus in part *c* and the xz-plane. [4]

* This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

12B

INSTRUCTIONS FOR RATING TWELFTH YEAR MATHEMATICS 12B (Solid Geometry)

Thursday, January 25, 1962 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20–25, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 8

(2) 7

(3) 5

(4) $4\pi r^2$

(5) 180π

(6) 10π

(7) $160\sqrt{3}$

(8) $6\sqrt{2}$

(9) 8

(10) 1

(11) $18\sqrt{2}$

(12) 7

(13) 90

(14) $36\sqrt{2}$

(15) $32\sqrt{3}$

(16) 1 : 64

(17) $\frac{25}{4}$

(18) 6

(19) $36\sqrt{3}$

(20) 3

(21) 2

(22) 2

(23) 4

(24) 1

(25) 1

(26) always

(27) never

(28) sometimes

(29) always

(30) always

[OVER]

Part II

Please refer to the Department's pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

(33) There are many ways of describing these loci. Each description should include shape and position. For instance, phrases such as the following should be allowed credit as indicated:

a (1) A cylindrical surface with radius d and axis n [2]
(2) Two planes parallel to P, one on either side and at a distance s from P [2]

b (1) Two parallel lines [2]
(2) A line [2]

c Two circles [2]

(34) *a* $s = \dfrac{2h}{\sqrt{2}\tan\theta}$ [4]

(35) *a* $\dfrac{10}{3}$, 10 [4]

b $\dfrac{320}{3}\pi$ [3]

c $\dfrac{4}{3}\sqrt{11}$ [3]

(36) *a* A sphere with center at the origin and a radius of 5 [2]

b A plane parallel to the yz-plane and intersecting the x-axis at (3, 0, 0) [2]

c A circle [2]

d (3, 0, 4) and (3, 0, —4) [4]