12B

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Monday, January 25, 1965 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 The dimensions of a rectangular parallelepiped are a, $3a$ and $5a$. Express the length of a diagonal of the parallelepiped in terms of a.

2 The plane angle of a dihedral angle is 60°. A point on the bisector of the dihedral angle is 16 units from the edge of the angle. Find the distance from this point to either face of the angle.

3 The altitude of a right circular cylinder is equal to the radius of its base. Find the ratio of the lateral area to the total area of the cylinder.

4 Find the number of square inches in the lateral area of the frustum of a regular hexagonal pyramid whose base edges are 4 inches and 6 inches and whose slant height is 10 inches.

5 An edge of a cube is 6. Find the area of a section made by a plane drawn through two diagonally opposite edges.

6 The lateral areas of two similar cones are 25 and 36. If the altitude of the smaller cone is h, express the altitude of the larger cone in terms of h.

7 The projection on a plane of a line segment 10 inches long is $5\sqrt{2}$ inches. Find the number of degrees in the angle formed by the line and the plane.

8 The volume of a pyramid is 81 cubic inches. A plane passing through the pyramid parallel to the base cuts off a pyramid whose volume is 3 cubic inches. Find the ratio of the area of the base of the larger pyramid to the area of the base of the smaller.

9 A prism and a pyramid have equal volumes and equal bases. If the altitude of the prism is h, express the altitude of the pyramid in terms of h.

10 The diameter of the base of a circular cone is 10 inches. Find the number of square inches in the area of a section which is parallel to the base and bisects the altitude of the cone.

11 Each side of a spherical triangle equals 80°. Find the number of spherical degrees in the area of its polar triangle.

12 Assuming that the earth's surface is a sphere, what fractional part of the earth's surface is included between the parallels 30° north and 30° south latitude?

13 The area of a great circle of a sphere is 12π square units. Find the volume of the sphere.

14 A cube is inscribed in a sphere whose radius is $2\sqrt{3}$ inches. Find the number of square inches in the surface of the cube.

15 A solid spherical ball 4 inches in diameter weighs 2 pounds. Find in pounds the weight of a solid ball of the same material 8 inches in diameter.

16 An isosceles right triangle is revolved through 180° about the altitude, drawn to the hypotenuse, as an axis. If a leg of the triangle is s, find the lateral area of the cone formed in terms of s.

[OVER]

17 The height of a frustum of a circular cone is 9 inches. If the radii of the bases are 3 inches and 5 inches, find the number of cubic inches in the volume of the frustum.

18 If a right circular cylinder is circumscribed about a sphere, find the ratio of the volume of the cylinder to the volume of the sphere.

19 Find the number of degrees in the angle of a lune if the area of the lune is $\frac{1}{8}$ the area of the sphere on which it is drawn.

Directions (20–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes the statement.

20 A cube and a regular octahedron have the same number of

(1) edges
(2) diagonals
(3) faces
(4) vertices

21 Two lines are always parallel if they

(1) do not intersect
(2) are perpendicular to the same plane
(3) are perpendicular to the same line
(4) are parallel to the same plane

22 Points A and B are 12 inches apart. The locus of points 8 inches from both A and B is

(1) a plane
(2) two points
(3) a circle
(4) two circles

23 A replacement for x which will make $V = x(B + B' + 4M)$ the formula for the volume of a prismatoid is

(1) h
(2) $\frac{h}{2}$
(3) $\frac{h}{3}$
(4) $\frac{h}{6}$

24 In an equilateral skew quadrilateral, the diagonals

(1) bisect each other
(2) do not intersect each other
(3) are perpendicular to each other
(4) are parallel to each other

25 The three face angles of a trihedral angle may be in the ratio

(1) 1:1:2
(2) 1:2:3
(3) 1:3:4
(4) 2:3:4

26 Two planes are always parallel if

(1) each contains one of two parallel lines
(2) they are perpendicular to the same line
(3) they are perpendicular to the same plane
(4) they are parallel to the same line

27 If a line l is oblique to a plane M, then the number of planes containing l and also perpendicular to M is

(1) 1
(2) 2
(3) 0
(4) infinite

28 Given regular square pyramid V-$ABCD$. The bisector of dihedral angle AB intersects VD at F and VC at E. Figure $ABEF$ is a

(1) square
(2) rectangle which is not a square
(3) rhombus which is not a square
(4) trapezoid

29 A right section of an oblique prism is a square whose area is a^2. If the lateral edge of the prism is 10, then the lateral area is

(1) $40a^2$
(2) $10a^2$
(3) $40a$
(4) $10a$

30 If the sum of two sides of a convex spherical polygon is 180°, then the sum of the remaining sides may be

(1) 150°
(2) 180°
(3) 210°
(4) 360°

Part II

Answer four questions from this part. Show all work unless otherwise directed.

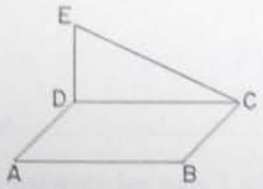
31 Prove *either* *a* or *b* but *not* both: [10]

a If two lines are parallel, every plane containing one of these lines, and only one, is parallel to the other.

OR

b If the first of two spherical triangles is the polar triangle of the second, then the second is the polar triangle of the first.

32 In the accompanying figure, $ABCD$ is a rectangle with ED perpendicular to the plane of rectangle $ABCD$. EC is drawn.



Prove: Angle ECB is a right angle. [10]

33 Planes M and N intersect in line l. P and R are two points on l.

a Describe fully the locus of points which are

(1) equidistant from M and N [2]
(2) equidistant from P and R [2]
(3) a given distance k from l [2]

b Indicate the correct completion for *each* of the following by writing the *number* 1, 2, 3 or 4 after the letters A and B on your answer paper. [4]

(A) The locus of points satisfying both (1) and (2) of *a* consists of

(1) two intersecting lines
(2) two parallel lines
(3) four points
(4) four lines

(B) The locus of points satisfying both (2) and (3) of *a* consists of

(1) two parallel lines
(2) four parallel lines
(3) a circle
(4) two circles

34 *a* The dimensions of a rectangular parallelepiped are in the ratio 2 : 5 : 6, and the volume is 20,320 cubic feet. Find the dimensions to the *nearest foot*. [5]

b The angles of a spherical triangle are 80°, 75° and 140°, respectively, and the radius of the sphere is 10 inches. Find in terms of π the number of square inches in the area of the spherical triangle. [5]

35 In the accompanying figure, a right circular cone is constructed on the base of a hemisphere. The surface of the hemisphere is equal to the lateral surface of the cone. Show that the volume (V) of the solid formed can be found by the formula $V = \frac{1}{3}\pi r^3(2 + \sqrt{3})$. [10]

*36 Answer *either* *a* or *b* but *not* both:

a In a spherical triangle ABC, $C = 90°$, $A = 110°$ and $b = 35°$. Find a to the *nearest degree*. [10]

OR

b (1) Find the x-, y- and z-intercepts of the plane whose equation is $4x - 5y - 10z = 20$. [3]

(2) Find in radical form the distance between the points (4,—1,6) and (—1,2,3). [2]

(3) Find the coordinates of the midpoint of the line segment joining the points (4,—1,6) and (—1,2,3). [2]

(4) Write an equation of the plane parallel to the x-axis and passing through the points (0,—3,0) and (0,0,2). [3]

* These questions are based on optional topics in the syllabus.

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ANSWER SHEET

Pupil..... Teacher.....

School.....

Name and author of textbook used.....

All of your answers for part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1.....	11.....	21.....
2.....	12.....	22.....
3.....	13.....	23.....
4.....	14.....	24.....
5.....	15.....	25.....
6.....	16.....	26.....
7.....	17.....	27.....
8.....	18.....	28.....
9.....	19.....	29.....
10.....	20.....	30.....

Total
Part I Score.....

Rater's
Initials.....

Your answers for part II should be placed on paper supplied by the school.

FOR TEACHERS ONLY

12B

SCORING KEY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 21–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $a\sqrt{35}$

(2) 8

(3) 1:2

(4) 300

(5) $36\sqrt{2}$

(6) $\frac{6h}{5}$

(7) 45

(8) 9:1

(9) $3h$

(10) $\frac{25\pi}{4}$

(11) 120

(12) $\frac{1}{2}$

(13) $32\pi\sqrt{3}$

(14) 96

(15) 16

(16) $\frac{\pi s^2\sqrt{2}}{2}$

(17) 147π

(18) 3:2

(19) 40

(20) 1

(21) 2

(22) 3

(23) 4

(24) 2

(25) 4

(26) 2

(27) 1

(28) 4

(29) 3

(30) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

33 *a* (1) two intersecting planes bisecting the dihedral angles formed by M and N [2]
(2) a plane perpendicular to segment PR at its midpoint [2]
(3) a cylindrical surface with radius k and axis l [2]

b *A* (1) [2]
B (3) [2]

34 *a* 14, 35, 42 [5]

b $\frac{575\pi}{9}$ [5]

36 *a* 122 [10]

b (1) 5, —4, —2 [3]

(2) $\sqrt{43}$ [2]

(3) $\left(\frac{3}{2}, \frac{1}{2}, \frac{9}{2}\right)$ [2]

(4) $2y - 3z + 6 = 0$ [3]