

The University of the State of New York

260TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Friday, June 22, 1934 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-15) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 If an edge of a regular tetrahedron is e , the total surface of the tetrahedron expressed in terms of e is Ans.....

2 If the perimeter of a right section of a prism is 4 inches and a lateral edge is 8 inches, the lateral area of the prism is ... square inches. Ans.....

3 If the height of a right circular cylinder is 5 and its lateral area is 30π , the length of the radius of the base is Ans.....

4 If a line segment 2 inches long meets its projection on a plane at an angle of 27° , the length of the projection, correct to the nearest tenth of an inch, is ... inches. Ans.....

5 Two sides of a spheric triangle are 50° and 20° . The third side is greater than 30° and less than ... degrees and may have any value between these limits. Ans.....

6 If the vertex angle of an isosceles spheric triangle is 70° and one of the base angles is 80° , the number of spheric degrees in the area of the triangle is Ans.....

7 If the surfaces of two spheres are in the ratio 4:25, the ratio of their volumes is Ans.....

8 The formula for the area S of a zone of height h on a sphere of radius r is $S = \dots$ Ans.....

9 On a sphere of radius 6 is a lune whose angle is 60° ; the area of the lune is [Answer may be left in terms of π .] Ans.....

10 If the radii of the upper and lower bases of a frustum of a cone of revolution are 3 feet and 5 feet and the area of the curved surface is 24π square feet, the slant height of the frustum is ... feet. Ans.....

11 The locus of points at a distance of 3 inches from each of two given points 5 inches apart is a Ans.....

12 If the polar distance of a small circle of a sphere is 30° and the radius of the sphere is 10, the radius of the small circle is Ans.....

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13 If the lateral area of a right circular cone is exactly twice the area of its base, the ratio between the radius of its base and its slant height is

Ans.

14 If a plane is passed parallel to the base of a pyramid and bisects the altitude of the pyramid, the volume of the original pyramid is exactly . . . times the volume of the pyramid cut off by the plane.

Ans.

15 If the height of a regular square pyramid is h and a base edge is $2e$, the lateral area of the pyramid, expressed as a function of h and e , is

Ans.

Directions (questions 16–20) — Indicate whether each of the following statements is *always* true, *sometimes* true or *never* true by writing the word *always*, *sometimes* or *never* on the dotted line at the right.

16 One and only one plane can be passed through any given point perpendicular to a given line.

Ans.

17 If two lines are parallel to the same plane, they are parallel to each other.

Ans.

18 Through a given point *only* one plane can be passed perpendicular to a given plane.

Ans.

19 If two spheric triangles drawn on the same sphere are mutually equiangular, they are congruent.

Ans.

20 If a plane is parallel to one line and perpendicular to another, the two lines lie in the same plane.

Ans.

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- Write at top of first page of answer paper to part II (a) name of school where you have studied,
(b) number of weeks and recitations a week in solid geometry.
The minimum time requirement is five recitations a week for half a school year.
Name the author of the textbook you have used in solid geometry.

Part II

Answer five questions from part II, including three questions from group I and two questions from group II.

Group I

Answer three questions from this group.

21 Prove that if a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [10]

22 Prove that if the first of two spheric triangles is the polar triangle of the second, then the second is the polar triangle of the first. [10]

23 Given the plane P and any two points A and B that do not lie in P but are on the same side of P .

a State completely the locus of points that are at a given distance d from plane P . [3]

b State completely the locus of points that are equally distant from points A and B . [3]

c State completely the locus of points that satisfy both conditions given in a and b . [2]

d Under what conditions, if any, would there be no points satisfying both conditions given in a and b ? [2]

24 Prove that if a plane is passed through one of the diagonals of a parallelogram, the perpendiculars to this plane from the end points of the other diagonal are equal. [10]

Group II

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Use $\pi = \frac{22}{7}$ unless otherwise stated.

25 How many cubic yards of dirt must be removed in the excavation of a tunnel 21 feet high and 80 feet long if the cross section of the tunnel is a semicircle? [10]

26 A sphere and a circular cone have the same volume. The radius of the base of the cone is 6 inches and its height is 16 inches. Find the radius of the sphere correct to the nearest tenth of an inch. [10]

27 An oblique prism has for its base a rectangle whose dimensions are a and b . The lateral edge c of the prism makes with its projection on the base an angle x .

a Derive a formula for the volume V of the prism in terms of a , b , c and angle x . [6]

b Find, correct to the nearest integer, the value of V if $a = 3.2$, $b = 4.7$, $c = 13.9$ and angle $x = 72^\circ$ [4]