

The University of the State of New York

269TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 17, 1937 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

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See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

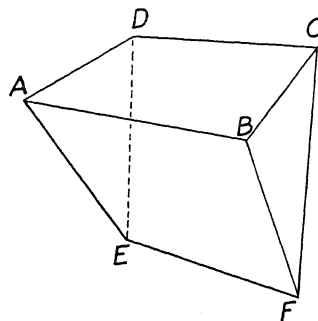
- 21 Prove that two lines perpendicular to the same plane are parallel. [10]
- 22 Prove that the sum of the face angles of any convex polyhedral angle is less than four right angles. [10]
- 23 Prove that if from the foot of a perpendicular to a plane a line is drawn at right angles to any line in the plane, the line drawn from the point of intersection formed to any point in the perpendicular is perpendicular to the line of the plane. [10]
- 24 Prove that if two sides of a spheric triangle are equal, the great circle arcs which bisect the angles opposite the equal sides and which are terminated by those sides are equal. [10]
- 25 Prove that if any two polyhedrons are circumscribed about equal spheres, the volumes of the two polyhedrons have the same ratio as their surfaces. [Suggestion: Draw lines from the vertices of the polyhedron to the center of the sphere.] [10]

Group III

Answer two questions from this group.

- 26 A zone and a spheric triangle drawn on the same sphere have equal areas. If the angles of the triangle are 70° , 80° , and 100° and the height of the zone is $5\frac{1}{2}$ feet, find the radius of the sphere. [10]
- 27 The outside diameter of a spheric shell made of a certain metal is 12 inches and the thickness is one inch. If the metal weighs 0.21 pounds per cubic inch, find, correct to the *nearest pound*, the weight of the shell. [Use $\pi = \frac{22}{7}$] [10]
- 28 A church spire has the form of a regular hexagonal pyramid whose base edge is 12 feet and whose lateral edge makes with the base edge an angle of 83° . Find, correct to the *nearest dollar*, the cost of painting the spire at 15 cents per square foot. [10]

- *29 The wedge, as shown in the accompanying figure, is a special case of the prismatoid. AC is a rectangle and EF is parallel to the plane of AC . Find the volume of the wedge if $AD = 8$, $AB = 12$, $EF = 10$, and the distance from EF to the plane of AC is 9.



[Use the formula for the volume of a prismatoid: $V = \frac{h}{6}(b + b' + 4m)$] [10]

* This question is based on one of the optional topics in the syllabus.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2 1/2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-11) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 Given any two lines not in the same plane; then one and only one plane can always be passed through one of these lines ... to the other. Ans.....

2 Two intersecting planes each tangent to a cylinder intersect in a line which is parallel to every ... of the cylinder. Ans.....

3 Two acute dihedral angles whose faces are parallel are Ans.....

4 The lateral area of a prism is equal to the product of a lateral edge and the perimeter of Ans.....

5 If the slant height of a frustum of a right circular cone is 6 inches and the radii of its bases are 4 inches and 2 inches, the lateral area of the frustum is ... square inches. [Answer may be left in terms of pi.] Ans.....

6 Two similar polyhedrons have corresponding diagonals which are in the ratio 2:3. If the total surface of the smaller polyhedron is 40 square inches, the total surface of the larger is ... square inches. Ans.....

7 If the sides of a spheric triangle contain 100°, 85° and 90°, the number of degrees in the largest angle of its polar triangle is Ans.....

8 The total surface of a cube is equal to exactly ... times the square of a diagonal of the cube. Ans.....

9 The base of a pyramid is an equilateral triangle of side a and the height of the pyramid is equal to a base edge. The volume of the pyramid expressed as a function of a is V = Ans.....

10 If a cone of revolution and a cylinder of revolution have equal heights and equal volumes, the radius of the base of the cone is to the radius of the base of the cylinder as sqrt(3) is to Ans.....

11 A plane passes through the center of two concentric spheres; the locus of points in the given plane and equidistant from the two spheres is a Ans.....

Directions (questions 12-15) — Indicate the correct answer to each of the following questions by writing on the dotted line at the right the letter a, b or c.

12 A lune whose angle is A is drawn on a sphere whose radius is r. If A is doubled, the area of the lune is multiplied by (a) 2, (b) 4 or (c) 8. Ans.....

13 An exterior angle of a spheric triangle is (a) equal to, (b) less than or (c) greater than, the sum of the two nonadjacent interior angles. Ans.....

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14 If the lateral area of a cylinder of revolution is one half its total area, the radius of its base is (a) equal to, (b) greater than or (c) less than, its height.

Ans.....

15 The number of points equally distant from the vertices of a triangle and, at the same time, at a given distance from the center of the circumscribed circle, is (a) one, (b) two or (c) infinite.

Ans.....

Directions (questions 16–20) — Indicate whether each of the following statements is *always*, *true*, *sometimes* true or *never* true by writing on the dotted line at the right the word *always*, *sometimes* or *never*.

16 The sum of the angles of a spheric triangle is greater than 180° and less than 540° .

Ans.....

17 The bases of a frustum of a pyramid are similar polygons.

Ans.....

18 The area of a section of a cone made by a plane parallel to the base and bisecting an element is one half the area of the base.

Ans.....

19 The volume of a parallelepiped is equal to the product of three concurrent edges.

Ans.....

20 If two opposite faces of a quadrangular prism are rectangles, the prism is a right prism.

Ans.....