

The University of the State of New York

291ST HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 22, 1944 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

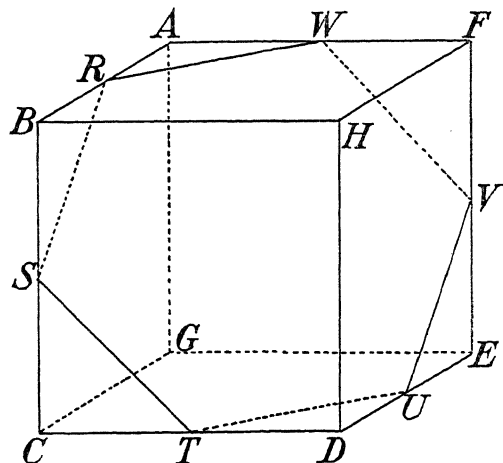
Part II

Answer three questions from part II.

- 21 Prove that if each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane. [10]
- 22 Given rectangle $ABCD$ with $AB = 8''$ and $BC = 6''$
- a Describe fully the locus of points $10''$ from vertex A . [2]
 - b Describe fully the locus of points equidistant from A, B, C and D . [3]
 - c How many points are there which satisfy the conditions given in both a and b? [1]
 - d How far from the plane of the rectangle are the points mentioned in c? [Answer may be left in radical form.] [4]
- 23 Prove that the sum of the angles of a spherical triangle is greater than 180° and less than 540° . [10]

24 Points R, S, T , etc. are the mid-points of the edges AB, BC, CD , etc. of the cube CF .

- a Prove that R is equidistant from the vertices G and H . [3]
- b Are S, T, U , etc. also equidistant from G and H ? [1]
- c Why are R, S, T, U , etc. coplanar? [3]
- d Prove that RS, ST, TU , etc. are equal. [1]
- e What additional fact must be shown in order to prove that hexagon $RSTUVW$ is regular? [2]

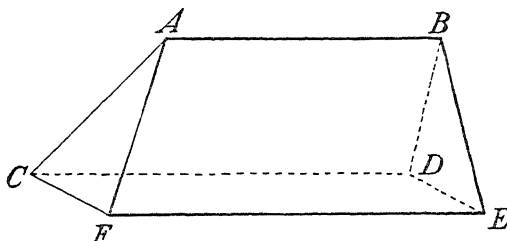


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*25 The figure at the right represents a wedge. $CDEF$ is a rectangle and AB is parallel to FE . Using the prismatoid formula:

$$V = \frac{h}{6} (B + B' + 4M),$$

find V if $CD = 8$, $CF = 4$, $AB = 6$ and the distance from AB to the plane of $CDEF$ is 3. [10]



Part III

Answer two questions from part III.

26 A tunnel whose cross section is a semicircle is to be constructed. The length of the tunnel is to be 108 feet and the diameter of the cross section 35 feet. A contractor estimates that the amount of earth to be removed in excavating for the tunnel will be twice the actual capacity of the tunnel. Allowing $2\frac{1}{2}$ cu. yd to the load, find the number of loads of earth which, according to this estimate, must be removed in constructing the tunnel. [Use $\pi = \frac{22}{7}$] [10]

27 A metal sphere 8.00 inches in diameter is melted and cast into a cube. Find, correct to the nearest tenth of an inch, the edge of the cube. [Use $\pi = 3.14$ and compute by means of logarithms.] [10]

28 The slant height of a frustum of a regular square pyramid makes with the lower base an angle θ . The lower base edge is m and the upper base edge is n .

a Show that the lateral area K of the frustum is given by the formula $K = \frac{m^2 - n^2}{\cos \theta}$ [7]

b Find K , correct to the nearest integer, if $m = 2.3$, $n = 1.4$ and $\theta = 58^\circ$ [3]

* This question is based on one of the optional topics in the syllabus and may be used in either part II or part III.

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Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–6) — Indicate whether each statement is true or false by writing the word *true* or the word *false* on the line at the right.

- | | |
|---|--------|
| 1 Lines parallel to the same plane are parallel to each other. | 1..... |
| 2 Planes perpendicular to the same plane are parallel. | 2..... |
| 3 A line parallel to each of two intersecting planes is parallel to their line of intersection. | 3..... |
| 4 The locus of points at a given distance from a given line is a plane. | 4..... |
| 5 The face angles of a trihedral angle may be 100° , 120° and 130° . | 5..... |
| 6 The sides of a spherical triangle may be 50° , 40° and 95° . | 6..... |

Directions (questions 7–9) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete each statement and write this word on the line at the right.

- | | |
|---|--------|
| 7 If the plane of a parallelogram is oblique to a given plane, the projection of the parallelogram on the given plane is ... a parallelogram. | 7..... |
| 8 If two spherical triangles on equal spheres have the three angles of one equal to the three angles of the other, the triangles are ... congruent. | 8..... |
| 9 The shortest distance between two points on the surface of the earth is ... an arc of a small circle of the earth. | 9..... |

Directions (questions 10–20) — Write the answer to *each* question on the line at the right.

- | | |
|---|---------|
| 10 The dimensions of a rectangular parallelepiped are 3, 4 and 12. Find a diagonal of the parallelepiped. | 10..... |
| 11 Express the volume of a circular cone in terms of its altitude h and the radius r of its base. | 11..... |
| 12 The radius of the base of a cone of revolution is 8 and its altitude is 6. Find the lateral area of the cone. [Answer may be left in terms of π .] | 12..... |
| 13 Find the volume of a regular triangular prism whose base edge is 4 and whose lateral edge is 10. [Answer may be left in radical form.] | 13..... |
| 14 The slant height of a regular hexagonal pyramid is equal to its base edge e . Express the lateral area of the pyramid in terms of e . | 14..... |
| 15 How far from the vertex of a pyramid whose altitude is 12 inches must a plane be passed parallel to the base so as to form a section whose area is $\frac{1}{3}$ the area of the base? | 15..... |
| 16 The volumes of two similar cylinders of revolution are in the ratio 1:64. Find the ratio of their total areas. | 16..... |

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17 The altitude of the North Frigid Zone is approximately 328 miles. The area of the zone is (a) less than 4 million square miles, (b) greater than 9 million square miles or (c) approximately 8 million square miles. Which is correct (a), (b) or (c)? [The radius of the earth is approximately 4000 miles.]

17.....

18 The angle of a lune is 15° and the radius of the sphere on which it is drawn is $3\frac{1}{2}$. Find the area of the lune. [Use $\pi = \frac{22}{7}$]

18.....

19 One angle of a spherical triangle is 85° . How many degrees are there in the opposite side of the polar triangle?

19.....

20 How many spherical degrees are there in the area of a spherical triangle whose angles are 100° , 70° and 80° ?

20.....