

The University of the State of New York

297TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 20, 1946—9.15 a. m., to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that the sum of any two face angles of a trihedral angle is greater than the third face angle. [10]

22 Prove that if a line in one face of a dihedral angle is parallel to a line in the other face, their plane is parallel to the edge of the dihedral angle. [10]

23 Prove that if a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle through these points. [10]

*24 A and A' are two fixed points d distance apart. Point P moves so that the sum of its distances from A and A' is a constant s where $s > d$.

a Represent by means of a drawing the locus of point P . [4]

b What is the name of this locus? [1]

c If d is allowed to become smaller and smaller until finally its value is zero, what does the locus become? [5]

*This question is based on one of the optional topics in the syllabus and may be used in either part II or part III.

Part III

Answer three questions from part III.

25 The angles of a spherical quadrilateral on a sphere of radius 15 inches are 80° , 110° , 125° and 95° .

a Find in square inches the area of the quadrilateral. [Answer may be left in terms of π .] [7]

b Find the angle of a lune on the same sphere which has the same area as the quadrilateral. [3]

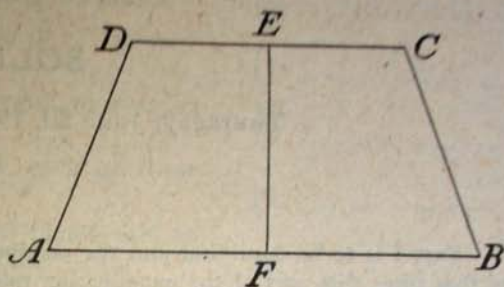
26 A regular hexagonal pyramid each side of whose base is 8 feet and each of whose lateral edges is 17 feet has the same volume as a right prism with a square base 6 feet on a side. Find, correct to the nearest foot, the height of the prism. [10]

27 A right circular cylinder of radius 12 inches is partly filled with water. When a sphere is completely immersed in the water in the cylinder, the surface of the water rises 4 inches. Find, correct to the nearest tenth of an inch, the radius of the sphere. [10]

28 Isosceles trapezoid $ABCD$ is revolved through 180° about EF , the perpendicular bisector of AB , as an axis. If $AB = b$ and $DC = b'$,

a Show that the formula for the lateral area S of the solid generated is

$$S = \frac{\pi}{4} \left(\frac{b^2 - b'^2}{\cos A} \right) \quad [6]$$



b Find S correct to the nearest integer when $b = 8$, $b' = 5$ and $A = 68^\circ$ [Use $\pi = 3.14$] [4]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-6) — Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

- 1 If line AB is perpendicular to plane m at point B , then AB is perpendicular to every line in plane m drawn through point B . 1.....
- 2 If two lines are perpendicular to each other, any plane containing one and only one of these lines is perpendicular to the other line. 2.....
- 3 If two lines are parallel, only one plane can be passed through one of these lines parallel to the other. 3.....
- 4 If one lateral face of a prism is a rectangle, then the other lateral faces are also rectangles. 4.....
- 5 The angles of a spherical triangle are determined if the three sides of the triangle are given. 5.....
- 6 If two planes are tangent to a cone, their intersection passes through the vertex of the cone. 6.....

Directions (questions 7-11) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

- 7 Two unequal line segments, oblique to a plane ... have equal projections on that plane. 7.....
- 8 If line a is oblique to plane P , it is ... possible to pass a plane containing a that is perpendicular to P . 8.....
- 9 The area of a zone of one base is ... equal to the product of its altitude and the circumference of its base. 9.....
- 10 The sum of the angles of a spherical triangle is ... equal to 180° . 10.....
- 11 The locus of points on a sphere and also equidistant from two points on the sphere is ... a great circle of the sphere. 11.....
- 12 The plane angle of a dihedral angle contains 64° . A point within the angle and equidistant from each plane is 6.0 inches from the edge of the dihedral angle. Find, correct to the *nearest tenth of an inch*, the distance from the point to either plane. 12.....
- 13 The area of the base of a pyramid is 36 sq. in. Find the area of the section made by a plane parallel to the base and bisecting the altitude. 13.....
- 14 The lateral area of an oblique prism is 180 sq. in. If a right section of the prism is a regular pentagon 4 in. on a side, find the length of a lateral edge of the prism. 14.....
- 15 The volume of a cube is 64 cu. in. Find the volume of the sphere inscribed in the cube. [Answer may be left in terms of π .] 15.....
- 16 If the edge of a regular tetrahedron is e , express its total area in terms of e . 16.....

17 Find the lateral area of the frustum of a regular hexagonal pyramid whose base edges are 3 and 4 and whose slant height is 7.

17.....

18 The lateral area and the volume of a right circular cylinder are numerically equal. Find the radius of the base.

18.....

19 The height of a right circular cone is equal to the radius of its base. If the volume of the cone is 72π cu. in., find the radius of the base of the cone.

19.....

20 The angles of a spherical triangle are 90° , 100° and 62° . Find the area of the triangle in spherical degrees.

20.....