SOLID GEOMETRY

Monday, June 15, 1959—1:15 to 4:15 p.m., **only**

Part 1

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. The perimeter of the base of a regular pyramid is 15 and the slant height is 6. Find the lateral area of the pyramid. 1...............

2. A right triangle has a base of 3 inches and an altitude of 7 inches. The triangle is revolved through 360° about the altitude to form a cone. Find the number of cubic inches in the volume of the cone. [Use the approximation $\pi = 22/7$.] 2...............

3. The diameter of the base of a right circular cylinder is equal to the altitude h of the cylinder. Express the total area of the cylinder in terms of h. 3...............

4. A plane is passed parallel to the base of a pyramid and 4 inches from its vertex. The area of the section made by the plane is 20 square inches. If the altitude of the pyramid is 10 inches, find the number of square inches in the base of the pyramid. 4...............

5. The area of the base of a right circular cone is 25π square inches and the length of the altitude of the cone is 3 inches. Find to the *nearest degree* the angle that an element makes with the base. 5...............

6. The base edges of the frustum of a regular square pyramid are 3 and 8. If the lateral area of the frustum is 132, find the slant height. 6...............

7. The radii of the bases of a frustum of a cone of revolution are 2 and 5, and the slant height is 6. Find the altitude of the frustum. 7...............

8. The diameter of a sphere is 3. Find the volume of the sphere to the *nearest tenth.* [Use the approximation $\pi = 3.14$.] 8...............

9. Similar solids have volumes of 320 and 5. If an edge of the larger solid is 8, find the corresponding edge of the smaller solid. 9...............

10. A zone on a sphere of radius $3\frac{1}{2}$ has an altitude of $\frac{1}{2}$. Find the area of the zone. [Use the approximation $\pi = 22/7$]. 10...............

Directions (11-15): Indicate the correct completion for *each* of the following by writing the letter *a*, *b*, *c* or *d* on the line at the right.

11. A rectangular solid with dimensions 6, x and y has a diagonal of 10. If x is to be an integer, the *largest* value that x can have is (*a*) 6 (*b*) 7 (*c*) 8 (*d*) 9 11.......

12. There is a regular polyhedron which has exactly (*a*) 4 vertices and 5 edges (*b*) 5 vertices and 10 edges (*c*) 6 vertices and 12 edges (*d*) 8 vertices and 16 edges 12........

13. If the area of a sphere is x, the area of a small circle of the sphere may be (*a*) $1/5x$ (*b*) $1/4x$ (*c*) $1/3x$ (*d*) $1/2x$ 13........

14. A convex polyhedral angle has five face angles, four of them being 60°, 70°, 80° and 90°. The fifth angle may be (*a*) 50° (*b*) 60° (*c*) 70° (*d*) 80° 14........

15. The angles of a spherical triangle may be (*a*) 40°, 50° and 80° (*b*) 150°, 180° and 210° (*c*) 100°, 200° and 300° (*d*) 90°, 90° and 90° 15........

Directions (16-20): If the blank space in each statement below is replaced by the word *always, sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

16. The projection of a triangle on a plane is .... a straight line segment. 16...............

17. The locus of points equidistant from the three vertices of a triangle is .... a plane. 17...............

18. Two lines perpendicular to the same line are .... parallel to each other. 18...............

19. On a given sphere, a spherical triangle is .... equal in area to a lune whose angle is half the spherical excess of the triangle. 19...............

20. A line which is parallel to the edge of a dihedral angle but which is not in either face of the dihedral angle is .... parallel to both faces of the dihedral angle. 20...............

Part II

Answer five questions from this part. Show all work unless otherwise directed.

21. Prove: Two lines perpendicular to the same plane are parallel. [10]

22. Prove: Every section of a sphere made by a plane is a circle. [10]

23. Line k is parallel to plane P. Prove that a plane perpendicular to k is also perpendicular to P. [10]

24. Given a plane V and a line k perpendicular to V at Q. A point P on k is at a distance d from Q. M is the midpoint of line segment PQ. Describe *fully* the locus of points

a. at a distance $\frac{1}{2}d$ from k [2]
b. at a distance $\frac{1}{2}d$ from V [2]
c. at a distance $\frac{1}{2}d$ from M [2]
d. that satisfy the conditions given in *a* and *b* [2]
e. that satisfy the conditions given in *b* and *c* [2]

25. In rectangle $ABCD$ with diagonal AC, let x represent the length of side AB and y the length of side BC. The rectangle is revolved through 360° about AB as an axis.

a. Find in terms of x and y the volume of the solid generated by triangle ABC. [3]

b. Find in terms of x and y the volume of the solid generated by triangle ADC. [5]

c. If there are 150 cubic inches in the volume generated by triangle ABC, how many cubic inches are in the volume generated by triangle ADC? [2]

26. A frustum of a cone of revolution is inscribed in a frustum of a regular pyramid whose bases are squares. Express in *simplest* form in terms of π the ratio of

a. the volume of the frustum of the cone to the volume of the frustum of the pyramid [$V = 1/3h\,(B_1 + B_2 + \sqrt{B_1B_2})$.] [5]

b. the lateral area of the frustum of the cone to the lateral area of the frustum of the pyramid [5]

27. A lune, an equilateral triangle and a zone are on a sphere of radius 10, and each has an area equal to one-fifth the area of the sphere.

a. Find the height of the zone. [2]

b. Find the angle of the lune. [3]

c. Find the number of degrees in a side of the polar triangle of the given equilateral triangle. [5]

28. A metal sphere is melted and recast into a hollow spherical shell whose outer radius is 277 centimeters. The radius of the hollow interior of the shell is equal to the radius of the original sphere. Find to the *nearest centimeter* the radius of the original sphere. [10]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS
12B (Solid Geometry)

Monday, June 15, 1959 — 1:15 to 4:15 p.m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in solid geometry by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in solid geometry. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. *Either one or both* may be substituted for *any one or two* of the questions on part II of the examination in solid geometry, including question 22, which concerns a theorem that is *not* among the required theorems of the twelfth year syllabus.

*29 In spherical triangle ABC, side $a = 37°$, side $b = 50°$ and angle $C = 90°$.

a Find side c to the *nearest degree.* [8]

b Using the given data, write an equation that could be used to find angle B. [2]

*30 A sphere has its center at the origin with point P (2, 6, 3) on its surface. The sphere intersects the positive portions of the coordinate axes x, y and z at points A, B and C, respectively.

a Write an equation of the plane through P parallel to the xz-plane. [2]

b Write an equation of the sphere. [4]

c Write an equation of the plane through A, B and C. [4]

FOR TEACHERS ONLY

SG

INSTRUCTIONS FOR RATING
SOLID GEOMETRY
and
TWELFTH YEAR MATHEMATICS
12B (Solid Geometry)

Monday, June 15, 1959 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 11–15, allow credit if the pupil has written the correct answer instead of the letter *a, b, c* or *d.*

(1) 45

(2) 66

(3) $\frac{3\pi h^2}{2}$

(4) 125

(5) 31

(6) 6

(7) $3\sqrt{3}$ *or* 5.2

(8) 14.1

(9) 2

(10) 11

(11) *b*

(12) *c*

(13) *a*

(14) *a*

(15) *d*

(16) sometimes

(17) never

(18) sometimes

(19) always

(20) always

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

Part II

24 There are many ways of describing these loci. Each description should include shape and position. For instance, phrases such as the following should be allowed credit as indicated:

a a cylindrical surface with radius $\frac{1}{2}d$ and axis k [2]
b two planes parallel to V, one on either side, and at a distance $\frac{1}{2}d$ [2]
c a spherical surface with radius $\frac{1}{2}d$ and center M [2]
d two circles, one in each of the planes described in *b*, with radius $\frac{1}{2}d$ and centers on k [2]
e a circle with radius $\frac{1}{2}d$ and center M, in a plane which is parallel to V [2]

25 *a* $\frac{1}{3}\pi\ xy^2$ [3]
b $\frac{2}{3}\pi\ xy^2$ [5]
c 300 [2]

26 *a* $\frac{\pi}{4}$ [5]

b $\frac{\pi}{4}$ [5]

27 *a* 4 [2]
b 72° [3]
c 72° [5]

28 220 [10]

Twelfth Year Mathematics (Solid Geometry)

29 *a* 59° [8]
b $\tan b = \sin a \tan B$ [2]

30 *a* $y = 6$ [2]
b $x^2 + y^2 + z^2 = 49$ [4]
c $x + y + z = 7$ [4]