The University of the State of New York
Regents High School Examination

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Monday, June 17, 1963 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 The surface area of a hemispherical dome is 100π square feet. Find the number of feet in the radius of the dome. 1.....

2 The base of a right prism is a hexagon whose side is 2 centimeters. If the height of the prism is 10 centimeters, find the number of cubic centimeters in its volume. 2.....

3 The length of an edge of a cube is 10. Find the length of the radius of the sphere which may be circumscribed about the cube. 3.....

4 Each base edge of a regular square pyramid is $5\sqrt{2}$ inches. Each lateral edge makes an angle of 45° with a diagonal of the base. Find the number of cubic inches in the volume of the pyramid. 4.....

5 A right circular cylinder with radius 5 is inscribed in a sphere with radius 13. Find the lateral surface of the cylinder. 5.....

6 A line segment CP which is 12 inches long makes an angle of 30° with plane M. Find the number of inches in the projection of CP on plane M. 6.....

7 A plane is passed 3 inches from the center of a sphere. If the diameter of the sphere is 10 inches, find the number of square inches in the *smaller* of the two zones formed. 7.....

8 The radii of the upper and lower bases of a frustum of a right circular cone are 2 and 3, respectively. If the slant height of the frustum is 2, find the lateral area of the frustum. 8.....

9 A sphere of radius 1 has the same volume as a right circular cylinder whose height is 3. Find the length of a radius of the cylinder. 9.....

10 The sides of a spherical triangle are 55°, 76° and 130°. Find the number of degrees in the sum of the angles of its polar triangle. 10.....

[OVER]

11 Point P is 8 inches from each face of a dihedral angle and 10 inches from the edge of the angle. Find to the *nearest degree* the number of degrees in the plane angle of the dihedral angle. 11.....

12 The area of a birectangular spherical triangle is 40 spherical degrees. Find the number of degrees in the third angle of the triangle. 12.....

13 Assuming the earth to be a sphere, what fractional part of the earth's surface lies north of the equator between the meridians 36° E and 72° E? 13.....

14 An isosceles right triangle with hypotenuse of length c is revolved about one of its legs as an axis. Express in terms of c the lateral area of the cone thus formed. 14.....

15 Find the number of faces of a regular polyhedron that has 8 vertices and 12 edges. 15.....

16 The altitude of a pyramid is 6. The area of its base is 18. Find the area of the section made by a plane parallel to the base at a distance of 2 from the base. 16.....

17 What is the perimeter of a lune on a sphere whose radius is 10 inches? 17.....

Directions (18–27): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

18 The ratio of the volumes of two circular cones with equal radii is 1:64. The ratio of their altitudes is (1) 1:4 (2) 1:8 (3) 1:64 (4) $1:64^3$ 18.....

19 The locus of points equidistant from the points on a circle and also at a fixed distance of 10 from the center of the circle is

(1) 2 points
(2) 2 lines
(3) 2 circles
(4) 2 planes

19.....

20 Parallel lines AB and CD intersect plane R, but are *not* perpendicular to R. If planes S and T are passed through AB and CD, respectively, so that $S \perp R$ and $T \perp R$, then S and T may

(1) intersect in a line perpendicular to R
(2) intersect in a line oblique to R
(3) intersect in a line which lies in R
(4) be parallel

20.....

21 The length of a diagonal of a regular octahedron of edge e is

(1) $\frac{e\sqrt{2}}{2}$ (2) e (3) $e\sqrt{2}$ (4) $e\sqrt{3}$ 21.....

22 The locus of points equidistant from two intersecting lines and also at a given distance from the plane of the lines is

(1) 2 points
(2) 2 lines
(3) 4 points
(4) 4 lines

22.....

23 Given a plane M and a line t. Which condition is *not* sufficient to determine one and only one plane through t perpendicular to M?

(1) t is oblique to M
(2) t is parallel to M
(3) t lies in M
(4) t is perpendicular to M

23.....

24 Two face angles of a trihedral angle are 60° and 90°. The third face angle may be (1) 30° (2) 90° (3) 150° (4) 210° 24.....

25 In isosceles spherical triangle ABC, side a = side b. If the number of degrees in angle C equals the number of degrees in side c, then the number of degrees in side a

(1) is less than 90
(2) is equal to 90
(3) is greater than 90
(4) cannot be determined from the data given

25.....

26 Cylinder A has radius r and altitude h. Cylinder B has radius $2r$ and altitude $\frac{h}{2}$. The volume of cylinder B, compared to the volume of cylinder A, is

(1) twice as great
(2) four times as great
(3) eight times as great
(4) the same

26.....

27 The altitude of a regular tetrahedron of edge 6 is (1) $2\sqrt{7}$ (2) $2\sqrt{6}$ (3) 3 (4) $3\sqrt{2}$

27.....

Directions (28–30): If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

28 Two dihedral angles whose corresponding faces are parallel are ... equal. 28.....

29 If a great circle passes through one pole of another circle, it ... passes through the other pole. 29.....

30 If two angles of a spherical triangle are complementary, the triangle is ... a right spherical triangle. 30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

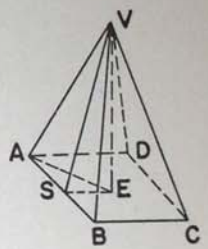
31 Prove *either* *a* or *b*: [10]

a Two lines perpendicular to the same plane are parallel.

OR

b The locus of points equally distant from two given points is the plane perpendicular to the line segment joining them at its midpoint.

32 In the accompanying figure, V–$ABCD$ is a regular square pyramid with altitude VE and slant height VS.



a Why is $AE > SE$? [2]

b *Prove* $\angle VSE > \angle VAE$. [Suggestion: On line AE lay off $TE = SE$ and draw VT.] [8]

33 Parallel planes M and N are 4 units apart. Line t is perpendicular to M. On t, P is a point which is 1 unit from M and 5 units from N.

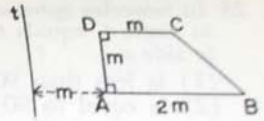
a Describe *fully* the locus of points equidistant from M and N. [3]

b Describe *fully* the locus of points which are k units from P. [3]

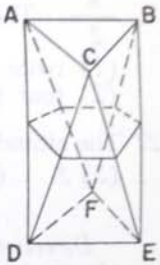
c Describe *fully* the locus of points satisfying the conditions in both *a* and *b* if $k = 5$. [4]

34 In the accompanying figure, $ABCD$ is a trapezoid. $\angle A = \angle D = 90°$, $AD = DC = m$ and $AB = 2m$. The line t which is in the plane of $ABCD$ is parallel to AD and m units from AD.

If the region $ABCD$ is revolved through 360° about line t, a solid is generated. Express in terms of m the volume of this solid. [10]



35 The accompanying figure represents one type of prismatoid. The bases are congruent equilateral triangles whose planes are parallel and 12 units apart. The lateral faces are congruent isosceles triangles. If $AB = 4$, find the volume of this prismatoid. [10]



*36 Answer *either* *a* or *b*:

a In spherical triangle RST, side r = side $s = 55°$ and side $t = 74°$. Find angle R to the *nearest degree*. [10]

OR

b (1) Write an equation of the plane which passes through the points A (2,0,0), B (0,—1,0) and C (0,0,4). [3]

(2) Which of the points P (4,0,4), Q (2,—2,4), R (2,1,2) and S (2,1,4) lies on the plane in part (1)? [3]

(3) Find the coordinates of the midpoint of segment PQ. [2]

(4) Find the length of segment PR. [2]

*This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

SCORING KEY

12B

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 18–27, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $5\sqrt{2}$

(2) $60\sqrt{3}$

(3) $5\sqrt{3}$

(4) $\frac{250}{3}$

(5) 240π

(6) $6\sqrt{3}$

(7) 20π

(8) 10π

(9) $\frac{2}{3}$

(10) 279

(11) 106

(12) 40

(13) $\frac{1}{20}$

(14) $\frac{\pi\sqrt{2}\,c^2}{2}$

(15) 6

(16) 8

(17) 20π

(18) 3

(19) 1

(20) 4

(21) 3

(22) 4

(23) 4

(24) 2

(25) 2

(26) 1

(27) 2

(28) sometimes

(29) always

(30) never

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

(33) *a* Another plane parallel to both M and N and midway between them [3]

b A spherical surface (sphere) with center P and radius k [3]

c A small circle of the sphere with radius 4 and center at the intersection of t with the plane described in part *a* [4]

(34) $\frac{16\pi m^3}{3}$ [10]

(35) $64\sqrt{3}$ [10]

(36) *a* 58 [10]

b (1) $2x - 4y + z = 4$ [3]

(2) S [3]

(3) (3,—1,4) [2]

(4) 3 [2]