The University of the State of New York

Regents High School Examination

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Tuesday, June 16, 1964 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 A plane 8 inches from the center of a sphere intersects the sphere in a circle whose area is 225π square inches. Find the number of inches in the radius of the sphere. 1.....

2 Find the number of inches in the altitude of a pyramid whose volume is 500 cubic inches and whose base is 50 square inches. 2.....

3 The lateral area of a frustum of a cone of revolution is 98π square inches, and the radii of the bases are 8 inches and 6 inches. Find the slant height of the frustum. 3.....

4 A point is 5 inches from each of two perpendicular planes. Find the number of inches in its distance from the line of intersection of the planes. 4.....

5 The base of a regular pyramid is a square 4 inches on a side, and the four lateral faces are equilateral triangles. Find the number of degrees in the angle which the lateral edge makes with the plane of the base. 5.....

6 A sphere of radius 9 inches and a sphere of radius 4 inches are tangent to each other at point P and tangent to a plane m at T and T'. Find the number of inches in TT'. 6.....

7 The dimensions of a rectangular parallelepiped are in the ratio 1:2:3, and a diagonal of the parallelepiped is $\sqrt{126}$ inches. Find the number of inches in the length of the smallest dimension. 7.....

8 The area of a birectangular spherical triangle equals one-twelfth of the area of the sphere. Find the number of degrees in the third angle of the triangle. 8.....

[over]

9 Find the number of square inches in the area of a zone of one base on a sphere of radius 10 inches if the polar distance of the base is equal to 60 degrees. 9.....

10 An equilateral spherical triangle, each of whose angles is 80 degrees, and a lune, whose angle is 50 degrees, are drawn on a sphere. Find the ratio of the area of the triangle to the area of the lune. 10.....

11 The area of the base of a pyramid is B square feet and the altitude of the pyramid is h feet. A section of the pyramid is parallel to the base and d feet from the vertex. Express in terms of B, d and h the area of the section. 11.....

12 If the generating triangle of a cone of revolution is an isosceles right triangle with hypotenuse $9\sqrt{2}$ inches, find the number of cubic inches in the volume of the cone. 12.....

13 Two regular tetrahedrons have volumes which are in the ratio of 1 : 8. If the total area of the smaller is S, express the total area of the larger in terms of S. 13.....

14 The sides of a spherical triangle are 88°, 94° and 80°. Find the number of degrees in the spherical excess of its polar triangle. 14.....

15 The radius of the base of a cylinder of revolution is 5 inches, and its axis is 12 inches. Find the number of square inches in the area of a section parallel to and 3 inches from the axis. 15.....

16 The height of a triangular prism is 15 inches, and its base is a right triangle whose legs are 3 inches and 4 inches. Find the number of cubic inches in the volume of the prism. 16.....

17 Find the number of cubic centimeters in the volume of a sphere whose area is 144π square centimeters. 17.....

18 Express the volume of a cube in terms of its diagonal, d. 18.....

Directions (19–30): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

19 The sum of the face angles of a trihedral angle is 240°. The angle which can *never* be a face angle of the trihedral angle is
(1) 1° (2) 60° (3) 90° (4) 120° 19.....

20 In spherical triangle ABC, the lengths of chords AB, BC and CA are $8\sqrt{2}$, $8\sqrt{2}$ and 8, respectively. If the radius of the sphere is 8, the sides of the spherical triangle are
(1) 45°, 90°, 90°
(2) 60°, 90°, 90°
(3) 45°, 45°, 90°
(4) 60°, 60°, 90° 20.....

21 If m is the perimeter of a section bisecting all the lateral edges of a regular pyramid, then the formula for the lateral area, S, of the pyramid in terms of m and its slant height, l, is
(1) $S = ml$
(2) $S = \frac{1}{2}\ ml$
(3) $S = \frac{1}{3}\ ml$
(4) $S = \frac{1}{4}\ ml$ 21.....

22 The fractional part of the earth's surface which is included between the meridians 20° East and 40° West is

(1) $\frac{1}{6}$ (2) $\frac{1}{9}$ (3) $\frac{1}{12}$ (4) $\frac{1}{18}$ 22.....

23 Line l is perpendicular to plane m at point A. Through A, a line b is drawn making an acute angle of $x°$ with line l. The locus of line b is a

(1) cylindrical surface
(2) plane surface
(3) spherical surface
(4) conical surface 23.....

24 A right circular cylinder is circumscribed about a sphere whose radius is 3 inches. The number of square inches in the lateral area of the cylinder is

(1) 9π
(2) 18π
(3) 36π
(4) 54π 24.....

25 The locus of points equidistant from the faces of a dihedral angle and equidistant from two points on the edge of the dihedral angle is a

(1) plane parallel to the edge
(2) plane perpendicular to the edge
(3) line parallel to the edge
(4) line perpendicular to the edge 25.....

26 A plane is determined if it passes through a given point and is

(1) perpendicular to a given plane
(2) parallel to each of two given parallel lines
(3) parallel to both of two given skew lines
(4) parallel to a given line 26.....

27 The perimeter of a convex spherical quadrilateral drawn on a sphere whose radius is 8 *must be* greater than zero and less than

(1) 4π
(2) 8π
(3) 12π
(4) 16π 27.....

28 In a regular octahedron, the sum of the number of edges and the number of vertices is

(1) 12
(2) 14
(3) 16
(4) 18 28.....

29 Three edges of a cube meet at one vertex of the cube. The section formed by passing a plane through the midpoints of these edges is a triangle whose angles are

(1) 30°, 60°, 90°
(2) 60°, 60°, 60°
(3) 45°, 45°, 90°
(4) 50°, 60°, 70° 29.....

30 Tangents are drawn from an external point to a sphere. The locus of the points of tangency is a

(1) small circle of the sphere
(2) great circle of the sphere
(3) conical surface
(4) zone of one base 30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b* but *not* both: [10]

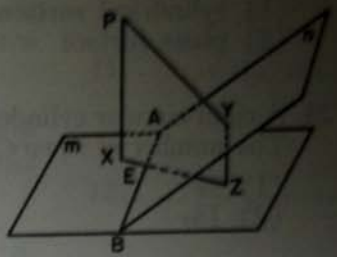
a Two lines perpendicular to the same plane are parallel.

OR

b A spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides produced if necessary.

32 In the accompanying figure, planes m and n intersect in line AB. From external point P, lines PX and PY are drawn perpendicular to planes m and n, respectively. From point Y a line YZ is drawn perpendicular to plane m, and XZ intersects AB in point E.

Prove that XZ is perpendicular to AB. [10]



33 Given points A and B which are 6 inches apart in plane m.

a Describe fully the locus of points in space at a distance r from plane m. [2]

b Describe fully the locus of points in space at a distance s from the line passing through A and B. [2]

c Describe fully the locus of points in space which are 5 inches from both point A and point B. [4]

d If the length of r is 2 inches and the length of s is 4 inches, then the locus common to *a*, *b* and *c* is (1) 2 points (2) 4 points (3) 2 lines (4) 4 lines. [Write the *number* 1, 2, 3 or 4 on your answer paper after the letter *d*.] [2]

34 In a triangle, the sides including an angle of 120° are m and $4m$. Express in terms of m the volume of the solid generated by rotating the triangle through 360° about the shortest side as an axis. [10]

35 In a frustum of a regular pyramid, the bases are squares with sides 6 inches and 12 inches, respectively. The lateral area of the frustum is half of the total area of the frustum. If x represents the number of inches in the slant height, then

a write an equation in terms of x which can be used to find the length of the slant height [4]

b solve the equation for x [2]

c find the number of inches in the height of the frustum [2]

d find the number of cubic inches in the volume of the frustum [2]

36 Derive a formula for the volume of a regular octahedron in terms of its edge e. [10]

*37 Answer *either* *a* or *b* but *not* both:

a Given a right triangle AOB with the origin O as the vertex of the right angle, A (6, 0, 0) and B (0, 8, 0).

(1) Write an equation of the locus of all points equidistant from the origin and point A. [2]

(2) Write an equation of the locus of all points equidistant from the origin and point B. [2]

(3) Describe the locus of all points equidistant from the vertices of the right triangle AOB. [3]

(4) Write an equation of the locus of all points 5 units from the origin. [3]

OR

b In isosceles spherical triangle RST, sides RS and ST are both equal to 54°, and side $RT = 42°$. Find angle R to the *nearest degree*. [10]

*This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

SCORING KEY

12B

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 19–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 17

(2) 30

(3) 7

(4) $5\sqrt{2}$

(5) 45

(6) 12

(7) 3

(8) 60

(9) 100π

(10) 3 : 5

(11) $\frac{Bd^2}{h^2}$

(12) 243π

(13) $4S$

(14) 98

(15) 96

(16) 90

(17) 288π

(18) $\frac{d^3\sqrt{3}}{9}$

(19) 4

(20) 2

(21) 1

(22) 1

(23) 4

(24) 3

(25) 4

(26) 3

(27) 4

(28) 4

(29) 2

(30) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

(33) *a* Two planes parallel to m, one on either side of m and r distance from it [2]

b A cylindrical surface with AB as axis and radius equal to r [2]

c A circle with center at midpoint of AB with radius 4 lying in the plane perpendicular to AB at its midpoint [4]

d 2 [2]

(34) $4\pi m^3$ [10]

(35) *a* $36x = 180$ [4]

b 5 [2]

c 4 [2]

d 336 [2]

(37) *a* (1) $x = 3$ [2]

(2) $y = 4$ [2]

(3) A line perpendicular to the xy-plane at the midpoint of line AB [3]

(4) $x^2 + y^2 + z^2 = 25$ [3]

b 74° [10]