# SOLID GEOMETRY

**Thursday,** August 22, 1940 — 8.30 to 11.30 a. m., only

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### Instructions

*Do not open this sheet until the signal is given.*

### Group I

*This group is to be done first and the maximum time allowed for it is one and one half hours.*

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

### Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) names of schools where you studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1940, (*d*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1940 is required.

## Fill in the following lines:

Name of school.............................................Name of pupil...............................................

**Detach this sheet and hand it in at the close of the one and one half hour period.**

### Group I

*Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.*

Directions (questions 1–9) — Indicate whether each statement is *always* true, *sometimes* true or *never* true by writing on the line at the right the word *always*, *sometimes* or *never*.

1 Two lines perpendicular to the same line are parallel to each other.

1.................

2 If a plane is perpendicular to both faces of a dihedral angle, it is perpendicular to the edge of the angle.

2.................

3 If two lines $a$ and $b$ are parallel and a third line $c$ is parallel to the plane of $a$ and $b$, then $c$ is parallel to both $a$ and $b$.

3.................

4 The face angles of a trihedral angle may be 105°, 120°, 135°.

4.................

5 If the projections of two lines on a plane are parallel, the lines are parallel.

5.................

6 Any side of a spheric triangle is a minor arc of a great circle.

6.................

7 Each of the angles of an equiangular spheric triangle may be equal to 60°.

7.................

8 If a right circular cylinder circumscribes a sphere, the lateral area of the cylinder is greater than the surface of the sphere.

8.................

9 The lateral area of a prism is equal to the product of the perimeter of the base and a lateral edge.

9.................

Directions (questions 10–20) — Write on the dotted line at the right of each question the expression which, when inserted in the corresponding blank, will make the statement true.

10 There can not be more than ... regular convex polyhedrons.

10.................

11 The sum of the sides of a convex spheric polygon is less than ... degrees.

11.................

12 A diagonal of a rectangular parallelepiped is 12 inches long and makes an angle of 34° with the base; the length of a diagonal of the base, correct to the *nearest tenth*, is ... inches.

12.................

13 If the lateral area and the volume of a cube are equal in numerical value, an edge of the cube must be ....

13.................

14 A lune whose angle is 90° has an area of 81$\pi$ square inches; the radius of the sphere on which this lune is drawn is ... inches.

14.................

15 The lateral area of a regular square pyramid whose lateral edge is 5 and whose base edge is 6 is ....

15.................

16 If the slant height of a cone of revolution is twice the radius of its base, the ratio of the lateral area to the area of the base is ....

16.................

[3]

17 The capacity in cubic feet of a tank in the form of a right circular cylinder whose height is 21 feet and the radius of whose base is 12 feet is ....    [Use $\pi = \frac{22}{7}$]

17................

18 If the radii of the upper and lower bases of a frustum of a cone of revolution are 3 feet and 5 feet and the area of the curved surface is $24\pi$ square feet, the slant height of the frustum is ... feet.

18................

19 The area of the base of a pyramid is 36; then the area of a section made by a plane parallel to the base and at a distance from the vertex equal to two thirds of the altitude is ....

19................

20 If the sides of a spheric triangle contain 100°, 85° and 90°, the number of degrees in the largest angle of its polar triangle is ....

20................

**See instructions for groups II and III on page 1.**

### Group II
**Answer three questions from this group.**

21 Prove that if a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [10]

22 Prove that in two polar triangles, each angle of one has the same measure as the supplement of the side lying opposite to it in the other. [10]

23 If any two lateral faces of a prism that are not parallel are rectangles, all of the lateral faces are rectangles and the prism is a right prism. [10]

24 State in full what the locus is in *each* of the following: [No proof is required.] [10]
    *a* The locus of points equidistant from the three vertices of a given triangle
    *b* The locus of points equidistant from the three edges of a given trihedral angle
    *c* The locus of all lines which make the same angle with a given line at a point on the line
    *d* The locus of the centers of all spheres that can be passed through two given points
    *e* The locus of points equidistant from two given parallel planes and a given distance from a straight line which is perpendicular to one of the planes

*25 The total areas of two similar tetrahedrons are in the ratio 16:25 and the volume of the first tetrahedron is 320 cubic inches.
    *a* What is the volume of the second tetrahedron? [5]
    *b* If the base of the first tetrahedron is 240 square inches, what is the corresponding altitude? [5]

### Group III
**Answer two questions from this group.**

26 A right circular cylinder 12 inches in diameter was partly filled with water. When an iron sphere was completely immersed in the water of the cylinder the surface of the water rose 2 inches. Find the radius of the sphere correct to the *nearest tenth of an inch.* [10]

27 A spheric triangle whose angles are 100°, 90° and 110° is drawn on a sphere whose radius is 6 inches. What must be the altitude of a zone on the same sphere for the area of the zone to equal the area of the spheric triangle? [10]

28 Triangle *ABC*, whose sides are 11, 13 and 20, revolves through 360° about side *BC* as an axis. Find the volume and the total surface of the figure generated. [Suggestion: $(AO)^2 = (13)^2 - x^2 = (20)^2 - (11 + x)^2$] [10]
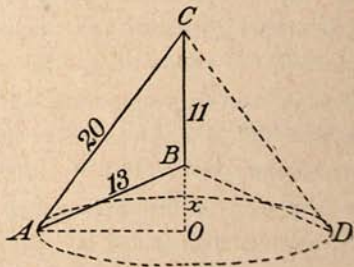


* This question is based on one of the optional topics in the syllabus.

[2]