The University of the State of New York

282D HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Friday, August 22, 1941 — 8.30 to 11.30 a. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II and III

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1941, (*d*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1941 is required.

See instructions for parts II and III on page 1.

Part II

Answer two questions from this part.

21 Prove that if each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane. [10]

22 *a* Describe the locus of points in plane MN that are equidistant from points A and B, 3″ and 4″ from MN and on the same side of MN. [$2\frac{1}{2}$]

b Describe the locus of all tangents to a sphere from an external point. [$2\frac{1}{2}$]

c AB is a fixed diameter of a sphere. What is the locus of the poles of all great circles passing through AB? [$2\frac{1}{2}$]

d A toy electric train runs on a circular track. What is the locus traced by a large number 1 printed on the side of the first car as the train makes a complete circuit of the track? [$2\frac{1}{2}$]

23 Prove that the line segments joining the mid-points of two pairs of opposite edges of a tetrahedron bisect each other. [10]

Part III

Answer three questions from this part.

24 A spheric triangle and a zone on the same sphere have equal areas. The angles of the triangle are 85°, 90° and 110° and the altitude of the zone is $\frac{7}{8}$ inch. Find the number of inches in the radius. [10]

25 The slant height of a frustum of a regular square pyramid makes an angle of 65° with the larger base. The sides of the bases are 3 inches and 6 inches. Find, correct to the *nearest cubic inch,* the volume of the frustum. [$V = \frac{1}{3}h(B_1 + B_2 + \sqrt{B_1B_2})$] [10]

26 A swimming tank is 65 feet long and 35 feet wide. At one end it is 4 feet deep and at the other it is 8 feet deep. Find the number of square feet in the inside surface of the tank. [10]

27 *a* The radius of the base of a right circular cone is r and the slant height of the cone makes an angle m with the base. Derive a formula for the volume V of the cone in terms of r and m. [5]

b Using the formula derived in answer to *a,* find, correct to the *nearest inch,* the radius of the cone if $V = 21.0$ cubic inches and $\tan m = 3$. [Use $\pi = 3.14$] [5]

Solid Geometry

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–9) — Indicate whether each statement is *true* or *false* by writing the word *true* or *false* on the dotted line at the right.

1 Two lines always determine a plane. 1.....

2 If three lines perpendicular to a given line at a given point are all parallel to a given plane, the given line is parallel to that plane. 2.....

3 If a line is parallel to each face of a dihedral angle, it is parallel to the edge of the dihedral angle. 3.....

4 The projection of a circle on a plane may be a straight line segment. 4.....

5 If three angles of a spheric quadrilateral are right angles, the fourth angle is obtuse. 5.....

6 Through either of two lines not in the same plane one plane and only one can be passed perpendicular to the other line. 6.....

7 Through a given point outside a plane one plane and only one can be passed parallel to the given plane. 7.....

8 If the edge of a cube is increased by a, the volume of the cube is increased by a^3. 8.....

9 In a pair of polar spheric triangles one triangle is always entirely included within the other. 9.....

Directions (questions 10–20) — Write on the dotted line at the right of each question the expression which, when inserted in the corresponding blank, will make the statement true.

10 In any convex polyhedron the sum of the face angles at any vertex is less than .... 10.....

11 If the lateral area of a right circular cylinder is increased by 24π when its altitude is increased by 3, the radius of the cylinder is .... 11.....

12 The area of a spheric triangle, one of whose angles is a right angle, is 30 spheric degrees; if one of the other angles of the triangle is 50°, the third angle contains ... degrees. 12.....

13 The altitude of a regular pyramid is 6 inches and its base is a square whose side is 9 inches. The area of a section made by a plane parallel to the base and at a distance of 2 inches from the vertex is ... square inches. 13.....

14 If the number of square inches in the area of a sphere equals the number of cubic inches in the volume of this sphere, then the radius of the sphere is ... inches. 14.....

15 If e is the edge of a regular tetrahedron, its slant height in terms of e is .... 15.....

16 A lune whose angle is 90° has an area of 81π square inches. The radius of the sphere on which the lune is drawn is ... inches. 16.....

17 The lateral area of a frustum of a right circular cone whose slant height is 3 and the radii of whose bases are 8 and 6 is .... [Answer may be left in terms of π.] 17.....

[OVER]

18 A plane passes through the center of a sphere whose radius is 6 inches. The locus of points one inch from the plane and one inch from the sphere consists of circles. The number of such circles is .... 18.....

19 The capacity in cubic feet of a tank in the form of a right circular cylinder whose height is 21 feet and the radius of whose base is 12 feet is ... cubic feet. [Use $\pi = \frac{22}{7}$] 19.....

20 If the lateral area of a right circular cone is exactly twice the area of its base, the ratio between the radius of the base and the slant height is .... 20.....