

The University of the State of New York

295TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, August 23, 1945 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in solid geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1945 or number and length in minutes of lessons taken in the summer of 1945 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For those pupils who have met the time requirement, the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1945 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that all the perpendiculars that can be drawn to a given line at a given point lie in the plane perpendicular to the line at that point. [10]

22 Prove that the diagonals of a parallelepiped meet in a point which is the midpoint of each diagonal. [10]

23 Prove that a spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides, produced if necessary. [10]

*24 Given plane MN and point P 4" from MN

a Name, or show by a drawing, the locus of points equally distant from MN and P . [5]

b Describe the locus of points 3" from MN . [3]

c What is the locus of points that satisfy the conditions in both a and b ? [2]

* This question is based on one of the optional topics in the syllabus.

SOLID GEOMETRY

Part III

Answer three questions from part III.

25 A pile of sand is in the form of a right circular cone the perimeter of whose base is 44 feet. The distance from the top of the pile to any point in the edge of the base is 11 feet. Find to the *nearest integer* the number of cubic yards of sand in the pile. [Use $\pi = \frac{22}{7}$] [10]

26 *a* Two angles of a spherical triangle are 100° and 125° and the area of the triangle is 34π . Find the third angle if the radius of the sphere is 6. [5]

b Show that the surface of the earth north of the 30° north parallel of latitude is one half the surface of the Northern Hemisphere. [5]

27 Gasoline was pumped under the English Channel to our army through pipes with an inner diameter of 3 inches and an outer diameter of $4\frac{5}{8}$ inches. Find, correct to the *nearest barrel*, the amount of gasoline a mile of one of these pipes would hold. [1 gal. = 231 cu. in., and $31\frac{1}{2}$ gal. = 1 bbl.] [10]

28 Two sides of a parallelogram are a and b , a being the longer, and the acute angle between them is θ . If the parallelogram is revolved about a as an axis, express in terms of a , b and θ the total area of the solid formed. [10]

SOLID GEOMETRY

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 If the faces of a convex polyhedron are congruent equilateral triangles, what is the greatest number of faces that can meet at one vertex? 1.....
- 2 The sides of a spherical triangle are 80° , 95° and 100° . Find the number of spherical degrees in the polar triangle. 2.....
- 3 An aquarium whose base is a rectangle $8''$ by $12''$ contains water to a height of $7''$. When a certain ornament is immersed in the water, the height of the water is $7\frac{1}{2}''$. Find the volume of the ornament. 3.....
- 4 The dimensions of a rectangular box are $8'' \times 8'' \times 4''$. What is the distance from any corner of the top to the diagonally opposite corner of the bottom of the box? 4.....
- 5 Find the lateral area of the frustum of a regular triangular pyramid if the slant height is 5 and the sides of the bases are 6 and 12. 5.....
- 6 What fractional part of the earth's surface lies between the meridians 15° E and 30° W longitude? 6.....
- 7 The altitude of a pyramid is $5''$ and the area of its base is 225 square inches. Find the area of the section of the pyramid made by a plane passed parallel to the base and $3''$ from it. 7.....
- 8 A line segment $20''$ long makes an angle of 20° with a plane. Find, correct to the *nearest inch*, the length of the line's projection on the plane. 8.....
- 9 The total area of a tin can is 40 sq. in. A tin can similar in shape and holding 8 times as much is to be made. What will be the total area of the new can? 9.....
- 10 A side of the base of a regular quadrangular pyramid is 3 and each lateral face makes an angle of 60° with the base. Find the lateral area of the pyramid. 10.....
- 11 The base of a pyramid is a right triangle with legs 4 and 5. The height of the pyramid is 9; find its volume. 11.....

Directions (questions 12-16) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b*, *c* or *d*.

- 12 If 4000 miles is used as the radius of the earth, the number of square miles in the earth's surface is most nearly (a) 100 million, (b) 200 million, (c) 250 million, (d) 270 million 12.....
- 13 If two spherical triangles are each symmetric to a third triangle, the two triangles must be (a) polar, (b) congruent, (c) symmetric 13.....
- 14 The locus of points at a given distance *d* from a line of unlimited length is (a) a line, (b) two parallel lines, (c) a cylindrical surface, (d) a cylinder 14.....
- 15 Given two skew lines and a point *P* not on either line. What is the greatest number of planes that can be passed through *P* parallel to the two lines? (a) 0, (b) 1, (c) 2, (d) none of these answers 15.....
- 16 In spherical triangle *ABC*, if angle *A* = 40° and angle *B* = 70° , then (a) *AB* is less than *AC*, (b) *AB* is equal to *AC*, (c) *AB* is greater than *AC* 16.....

SOLID GEOMETRY

Directions (questions 17–20) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

17 If the plane angles of two dihedral angles have their sides parallel, the dihedral angles are ... equal. 17.....

18 The section of a circular cone made by a plane cutting all the elements is ... a circle. 18.....

19 Two lines perpendicular to the same line are ... perpendicular to each other. 19.....

20 Two intersecting planes R and S are perpendicular to a third plane T . The three planes have point P in common. If at P a perpendicular to plane T is erected, this perpendicular ... lies in both R and S . 20.....