

The University of the State of New York

304TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Tuesday, August 24, 1948 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in solid geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1948 or number and length in minutes of lessons taken in the summer of 1948 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement, the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1948 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that all the perpendiculars that can be drawn to a given line at a given point lie in the plane perpendicular to the line at the point. [10]

22 Planes P and Q are parallel and line a , not in P , is parallel to Q . Prove that a is parallel to P . [10]

23 Prove that if the first of two spherical triangles is the polar triangle of the second, then the second is the polar triangle of the first. [10]

*24 Given plane P and line l perpendicular to P . Point M is on line l and 2 inches from P .

a Represent by a drawing the locus of points equidistant from M and P . [4]

b The locus of points 17 inches from both M and P is a circle. Give the position of the center of this circle and the length of its radius. [2, 4]

* This question is based on one of the optional topics in the syllabus and may be used in either part II or part III.

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Part III

Answer three questions from part III.

25 The angles of a spherical triangle are 74° , 88° and 95° . The area of the triangle is 193.6 square inches.

a Find the radius of the sphere on which the triangle is drawn. [Use $\pi = \frac{22}{7}$] [7]

b Find the volume of the sphere. [Answer may be left in terms of π .] [3]

26 The diameter of a closed cylindrical tank is 2 feet and its height is 8 feet, dimensions being taken on the outside. The thickness of the tank is .045 feet. Find to the *nearest gallon* the capacity of the tank. [$7\frac{1}{2}$ gal. = 1 cu. ft] [10]

27 A sphere is inscribed in a right circular cylinder and a right circular cone is inscribed in the sphere. The diameter of the base of the cone is equal to its slant height. The radius of the sphere is r .

a Express the altitude of the cylinder and the radius of the base of the cone in terms of r . [1, 2]

b Show that the total areas of the three solids are in the ratio 24:16:9 [7]

28 The base edge of a regular pentagonal pyramid is a and its altitude is h .

a Derive a formula for the volume V of the pyramid in terms of a and h . [5]

b Find V to the *nearest integer* if $a = 2.3$ and $h = 15.4$ [5]

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Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-7) — If the statement is always true write *true* on the line at the right; if it is not always true write *false*.

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|---|---------|
| 1 If a plane is perpendicular to the edge of a dihedral angle, it is perpendicular to each face of the dihedral angle. | 1..... |
| 2 A plane is determined if it passes through a given point and is parallel to each of two nonparallel lines. | 2..... |
| 3 The projection of a triangle on a plane is a triangle. | 3..... |
| 4 The locus of points equidistant from two intersecting planes and at a given distance from their line of intersection consists of four parallel lines. | 4..... |
| 5 A sphere can be circumscribed about any parallelepiped. | 5..... |
| 6 The sum of the angles of any convex spherical polygon of n sides is less than n straight angles. | 6..... |
| 7 The polar distance of any point on a small circle of a sphere whose radius is r is less than $\frac{\pi r}{2}$. | 7..... |
| 8 A line segment is 8 inches long and its projection on a plane is 5 inches. Find to the <i>nearest degree</i> the inclination of the line to the plane. | 8..... |
| 9 How far from the vertex of a pyramid whose altitude is 12 inches must a plane be passed so that the area of the section thus formed shall be equal to $\frac{1}{3}$ the area of the base? | 9..... |
| 10 The perimeter of a right section of a prism is 14 and its lateral edge is 5. Find the lateral area of the prism. | 10..... |
| 11 The base of a prism is an equilateral triangle. Its base edge is 4 and its altitude is 10. Find the volume of the prism. [Answer may be left in radical form.] | 11..... |
| 12 The base edge of a regular hexagonal pyramid is 3 and its slant height is 8. Find its lateral area. | 12..... |
| 13 The volumes of two similar cones of revolution are in the ratio 1 : 64. Find the ratio of their altitudes. | 13..... |
| 14 The radii of the bases of a frustum of a cone of revolution are 7 and 3 and the slant height is 5. Find the lateral area of the frustum. [Answer may be left in terms of π .] | 14..... |
| 15 The altitude of a zone is 3 and the radius of the sphere on which it is drawn is 12. Find the area of the zone. [Answer may be left in terms of π .] | 15..... |
| 16 What fractional part of the earth's surface is included between the meridians of 7° East Longitude and 8° West Longitude? | 16..... |
| 17 The sides of a spherical triangle are 70° , 80° and 90° . Find in spherical degrees the area of its polar triangle. | 17..... |

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Directions (questions 18–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

18 The face angles of a trihedral angle may be (a) 25° , 35° , 40°
(b) 26° , 52° , 24° (c) 130° , 110° , 120° 18.....

19 There is a regular polyhedron which has (a) 4 vertices and 8 edges
(b) 6 vertices and 12 edges (c) 8 vertices and 6 edges 19.....

20 Two spherical triangles on the same sphere are always congruent or symmetric if the following parts of one are equal to the corresponding parts of the other:
(a) any two sides and an angle (b) any two angles and a side
(c) the three angles 20.....