The University of the State of New York

316TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, August 21, 1952 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1952 or number and length in minutes of lessons taken in the summer of 1952 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1952 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that if each of two intersecting planes is perpendicular to a third plane, their line of intersection is also perpendicular to the third plane. [10]

22 Unequal line segments AB and CD are parallel to each other and are oblique to a given plane. Their projections on the plane are $A'B'$ and $C'D'$ respectively. Prove: $AB \times C'D' = CD \times A'B'$ [10]

23 Prove that if a point on a sphere is at a quadrant's distance from each of two other points, not the extremities of a diameter, it is the pole of the great circle through these points. [10]

24 Given triangular pyramid $ABCD$ with P a point on edge AB such that the plane determined by P and edge DC bisects the pyramid. Prove that P is the mid-point of AB. [10]

Part III

Answer three questions from part III.

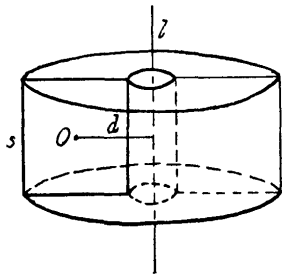
25 A trough has the form of a right prism. Each end of the trough is an isosceles trapezoid whose lower base is 8 inches, whose upper base is 16 inches and whose height is 12 inches. If the trough is $5\frac{1}{2}$ feet long, find to the *nearest gallon* the capacity of the trough. [1 gal = 231 cu. in.] [10]

26 The radius of the base of a right circular cone is r and an element of the cone makes with the base an angle θ. The lateral area of the cone is $1\frac{1}{4}$ times the area of the base.

a Find θ to the *nearest degree.* [4]

b Find the volume of the cone to the *nearest tenth of a cubic inch* if $r = 2.63$ inches. [Use $\pi = 3.14$] [6]

27 A square whose side is s is revolved through 360° about an axis l parallel to a side of the square, as shown in the drawing. The distance from O, the center of the square, to l is d. Show that the volume of the resulting solid is equal to the area of the square multiplied by the circumference of the circle whose radius is d. [10]



28 The perimeter of a triangle on a sphere whose radius is r is $p°$.

a If the area of the polar triangle is K, show that r is given by the formula

$$r = \sqrt{\frac{180\,K}{\pi(360 - p)}} \qquad [6]$$

b Using the formula given in *a*, find r to the *nearest tenth of an inch* when $K = 25$ square inches and $p = 150°$. [Use $\pi = \frac{22}{7}$] [4]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 The base edge of a regular square prism is e and its altitude is 3 e. Express its *total* area in terms of e. 1.....

2 Find the diagonal of a cube whose edge is 10. [Answer may be left in radical form.] 2.....

3 The slant height of a regular hexagonal pyramid is s and its base edge is e. Express its lateral area in terms of s and e. 3.....

4 Find the lateral area of a frustum of a cone of revolution if the radii of its bases are 2 and 5 and its slant height is 4. [Answer may be left in terms of π.] 4.....

5 A zone is drawn on a sphere whose radius is r. If the altitude of the zone is $\frac{1}{2}r$, express the area of the zone in terms of r. [Answer may be left in terms of π.] 5.....

6 If the area of a lune is $\frac{1}{30}$ the area of the sphere on which it is drawn, how many degrees are there in the angle of the lune? 6.....

7 Find the volume of a prism if a right section of the prism is a rectangle 3 inches by 8 inches and its lateral edge is 10 inches. 7.....

8 The base edge of a regular square pyramid is 6 and its slant height is 5. Find its volume. 8.....

9 The circumference of a great circle of a sphere is 6π. Find the volume of the sphere. [Answer may be left in terms of π.] 9.....

10 The altitude of a pyramid is 9 inches and its base is a right triangle whose legs are 6 inches and 12 inches. A plane is passed parallel to the base of the pyramid and 3 inches from the vertex. Find the area of the upper base of the frustum thus formed. 10.....

11 The areas of two regular tetrahedrons are in the ratio 1 : 4. If the volume of the smaller tetrahedron is V, express in terms of V the volume of the larger. 11.....

Directions (12–14) — Indicate the correct completion for each of the following by writing on the line at the right the letter a, b or c.

12 A sphere is inscribed in a right circular cylinder. If S represents the area of the sphere and S' the lateral area of the cylinder, then S is (a) greater than S' (b) equal to S' (c) less than S' 12.....

[OVER]

13 Given line l and point P on l. The locus of points at a given distance d from l and also at a given distance d' from P, is a circle if d is (*a*) greater than d' (*b*) equal to d' (*c*) less than d' 13.....

14 The angles of a spherical triangle may be
(*a*) 60°, 70°, 50° (*b*) 110°, 100°, 120° (*c*) 120°, 120°, 60° 14.....

Directions (15–20) — If the blank space in each statement is replaced by one of the words *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

15 If, in spherical triangle ABC, $AB = BC$, then the bisector of angle B ... divides the triangle into two congruent triangles. 15.....

16 If the three face angles of a trihedral angle are equal to the three face angles of another trihedral angle, then the trihedral angles are ... congruent. 16.....

17 If lines a and b are coplanar and are parallel to plane Q, then the plane determined by a and b is ... parallel to Q. 17.....

18 If lines a and b are not coplanar, then it is ... possible to pass more than one plane through a parallel to b. 18.....

19 If a and b are two intersecting lines in plane P and if P is parallel to plane Q, then the locus of points equidistant from a and b and also equidistant from P and Q ... consists of two intersecting lines. 19.....

20 The plane which passes through two diagonally opposite edges of a rectangular parallelepiped ... divides it into two congruent triangular prisms. 20.....