The University of the State of New York

319TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, August 20, 1953 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1953 or number and length in minutes of lessons taken in the summer of 1953 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1953 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that if two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]

22 Prove that the sum of the angles of a spherical triangle is greater than 180° and less than 540°. [10]

23 Given line l perpendicular to the plane of a circle at its center O. Prove that

a any point, P, on l is equidistant from all points on the circle [4]

b any point, P', not on l is not equidistant from all points on the circle. [*Suggestion:* Pass the plane determined by l and P', cutting the circle in diameter AB, and prove AP' and BP' are unequal.] [6]

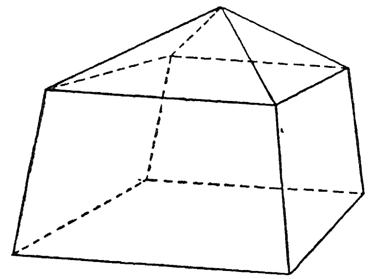
*24 The base of a truncated right prism is a right triangle whose legs are a and b. The lateral edges of the truncated prism are r, s and t. Using the prismatoid formula $V = \frac{h}{6}(B + B' + 4m)$, show that the volume of the prism is equal to $\frac{ab}{6}(r + s + t)$. [*Suggestion:* Let the base, B, of the prismatoid be a trapezoid whose parallel sides are two of the lateral edges and whose altitude is one of the legs of the right triangle.] [10]

* This question is based upon one of the optional topics in the syllabus.

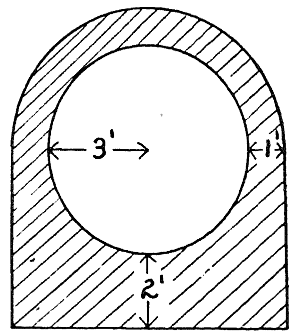
Part III

Answer three questions from part III.

25 A tent has the form of a frustum of a regular square pyramid surmounted by a regular pyramid, as shown in the drawing. The lower base edge of the frustum is 12 feet and its slant height is 7 feet. The base edge of the pyramid is 10 feet and its slant height is 6 feet. Find to the *nearest pound* the weight of the tent including the floor if the entire tent is made of canvas weighing 1.8 pounds per square yard. [10]



26 The accompanying drawing represents the cross section of a concrete conduit. Using the dimensions given in the drawing, find to the *nearest ten cubic yards* the amount of concrete necessary to construct this conduit if it is to be 300 feet long. [Use $\pi = \frac{22}{7}$ and make no allowance for waste.] [10]



27 Two of the angles of a spherical triangle on a sphere whose radius is 21 inches are 72° and 83°. The area of the triangle is 462 square inches. Using $\pi = \frac{22}{7}$, find

a the third angle of the triangle [6]

b the perimeter (in inches) of the polar triangle [4]

28 *V-ABCD* is a pyramid whose base is a rhombus. Diagonal *AC* of the base is 14.2, diagonal *BD* is 9.3, edge *VA* is 8.5, and the angle formed by *VA* and its projection on the base is 75°. Find to the *nearest integer* the volume of the pyramid. [10]

Solid Geometry

Fill in the following lines:

Name of pupil.. Name of school..

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

1 The right section of an oblique prism is a triangle whose perimeter is 10. A lateral edge of the prism is 20. Find its lateral area. 1................

2 The radius of the base of a cylinder of revolution is r and the altitude of the cylinder is $2r$. Express the lateral area of the cylinder in terms of r. 2................

3 The lateral area of a cone of revolution is 3 times the area of its base. Express as a common fraction the cosine of the angle at which the slant height is inclined to the base. 3................

4 The area of a sphere is 4π. Find the area of a zone of this sphere if its altitude is 0.5. [Answer may be left in terms of π.] 4................

5 The base of a parallelepiped is a rectangle 4 by 15. A lateral edge of the parallelepiped is 6 and is inclined to the base at an angle of 45°. Find its volume. [Answer may be left in radical form.] 5................

6 The volume of a right circular cone is 550 cubic inches and its altitude is 21 inches. Find the length in inches of the radius of its base. [Use $\pi = \frac{22}{7}$.] 6................

7 Express the volume of a sphere in terms of its diameter d. 7................

8 Two parallelepipeds have equal altitudes and their bases are in the ratio 5 : 4. Find the ratio of the volume of the larger parallelepiped to the volume of the smaller. 8................

9 The sum of the angles of a spherical triangle drawn on a sphere whose radius is 4 inches is equal to the sum of the angles of a triangle drawn on a sphere whose radius is 5 inches. Find the ratio of the area of the smaller triangle to that of the larger. 9................

10 The base of a pyramid is 16 square feet. The section of the pyramid formed by the plane parallel to the base and 2 feet from the vertex of the pyramid is 9 square feet. Find in feet the altitude of the pyramid. 10................

Directions (11–15): Indicate the correct completion for each of the following statements by writing on the line at the right the letter *a*, *b* or *c*.

11 A plane is determined if it passes through a given point and is (*a*) perpendicular to a given plane (*b*) parallel to each of two given parallel lines (*c*) perpendicular to a given line 11......

12 The locus of points equidistant from two given parallel planes and at a given distance from a given point midway between the two given planes (*a*) may be two points (*b*) is a circle (*c*) may be two circles 12......

13 The sum of the angles of a spherical quadrilateral may be (*a*) 360° (*b*) 700° (*c*) 750° 13......

14 The area of a lune is to the area of the sphere on which it is drawn as the number of degrees in the angle of the lune is to (*a*) 180 (*b*) 360 (*c*) 720 14......

15 In any regular tetrahedron of edge e (*a*) each dihedral angle is equal to 60° (*b*) the total area of the tetrahedron is equal to $e^2\sqrt{3}$ (*c*) the altitude of the tetrahedron is equal to $\frac{e}{2}\sqrt{3}$ 15......

Directions (16–20): In each of the following if the statement is *always* true, write *true* on the line at the right; if it is *not always* true, write *false.*

16 If two lines are not in the same plane, one plane and only one can be passed through one of these lines parallel to the other. 16................

17 The locus of points equidistant from two given intersecting planes consists of two planes. 17................

18 The sum of the face angles of a trihedral angle is less than 360° and the sum of any two face angles is greater than the third. 18................

19 If lines AA', BB', and CC' intersect at O, trihedral angles $O\text{-}ABC$ and $O\text{-}A'B'C'$ are congruent. 19................

20 If two of the sides of a triangle drawn on a sphere whose radius is r are each $\frac{\pi r}{2}$ and the third side is not πr, then one vertex of the triangle is the pole of the great circle passing through the other two. 20................