

The University of the State of New York

328TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Wednesday, August 22, 1956—12 m. to 3 p.m., only

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in solid geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1956.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1956 is required.

Part II

Answer two questions from this part.

21 Prove: A line perpendicular to one of two parallel planes is perpendicular to the other also. [10]

22 Prove: The sum of the angles of a spherical triangle is greater than  $180^\circ$  and less than  $540^\circ$ . [10]

23 Pyramid  $O-ABCD$  is a regular square pyramid with  $R$  the midpoint of  $OA$  and  $S$  the midpoint of  $OB$ . Prove that  $RSCD$  is an isosceles trapezoid. [10]

24 a Planes  $M$  and  $N$  intersect in line  $x$ . Planes  $R$  and  $S$  are parallel to each other and are perpendicular to  $x$ .

(1) Describe fully the locus of points at a given distance  $d$  from  $x$ . [2]

(2) Describe fully the locus of points equally distant from  $M$  and  $N$ . [2]

(3) Describe fully the locus of points equally distant from  $R$  and  $S$ . [2]

b Indicate the correct completion for each of the following statements by writing the letter  $a$ ,  $b$  or  $c$  after the proper number on your answer paper. [2, 2]

(4) The locus of points common to (1) and (2) above consists of (a) 2 parallel lines  
(b) 4 parallel lines (c) 8 parallel lines

(5) The locus of points common to (2) and (3) above consists of (a) one pair of intersecting lines  
(b) two pairs of intersecting lines (c) three pairs of intersecting lines

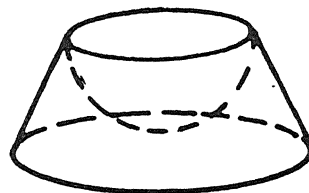
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Part III

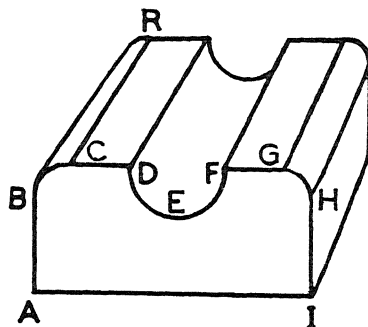
Answer three questions from this part. Show all work.

25 The sum of the sides of a spherical triangle is  $300^\circ$ . Find, to the *nearest tenth of a square inch*, the area of the polar triangle if the diameter of the sphere is 11 inches. [Use  $\pi = \frac{22}{7}$ .] [10]

26 An ashtray is a solid in the form of a frustum of a right circular cone with a hemisphere cut out of it, as shown in the diagram at the right. The diameters of the bases are 3 inches and 5 inches, and an element makes an angle of  $60^\circ$  with the lower base. Chromium plating costs \$1.54 a square foot. Find to the *nearest cent* the cost of chromium plating the exposed surface of the ashtray, *not* including the lower base. [Use  $\pi = \frac{22}{7}$ .] [10]



27 The figure at the right shows a metal casting. Angles  $A$  and  $I$  are right angles,  $DEF$  is a semicircle,  $BC$  and  $GH$  are quadrants,  $CD$  and  $FG$  are equal segments parallel to  $AI$ .  $AB$  is 3 inches, the radius of the quadrants  $BC$  and  $GH$  is  $1\frac{1}{2}$  inches,  $DF$  is 3 inches,  $AI$  is 9 inches and  $CR$  is 8 inches. The weight of the casting is 98 pounds. Find, to the *nearest hundredth of a pound*, the weight of 1 cubic inch of the metal of which it is made. [10]



28 The altitude of a regular tetrahedron is  $h$ .

a Show that an edge  $e$  of the tetrahedron equals  $\frac{h\sqrt{6}}{2}$ . [3]

b Show that the volume in terms of  $h$  is  $\frac{h^3\sqrt{3}}{8}$ . [2]

c If  $h = 4.97$ , find  $V$  to the *nearest tenth*. [5]

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Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of  $\pi$  or in radical form.

- 1 A right section of a prism is a regular pentagon. If a side of the right section is 3 and a lateral edge is 7, find the lateral area. 1.....
- 2 Find the volume of a circular cylinder whose radius is 4 and whose altitude is  $3\frac{1}{2}$ . 2.....
- 3 Express the altitude of a circular cone in terms of  $r$ , if the volume of the cone is  $\pi r^3$  and the radius of its base is  $r$ . 3.....
- 4 A right triangle whose legs are 6 and 8 is revolved through  $360^\circ$  about side 8 as an axis. Find the lateral area of the cone generated. 4.....
- 5 The base edge of a regular square pyramid is 8 and the altitude is 3. Find the lateral area of the pyramid. 5.....
- 6 Find the volume of a sphere whose diameter is 4. 6.....
- 7 The area of a lune in spherical degrees is 10. Find the number of degrees in an angle of the lune. 7.....
- 8 The weights of two solid spheres are in the ratio of 1:8. If the spheres are made of the same material and if the radius of the smaller is 3, find the radius of the larger. 8.....
- 9 A zone is drawn on a sphere whose radius is 6. If the altitude of the zone is 4, find its area. 9.....
- 10 A line 8 inches long is inclined to a plane at an angle of  $24^\circ$ . Find the length of the projection of this line on the plane to the *nearest tenth of an inch*. 10.....
- 11 The area of the base of a cone is 54 square inches and the altitude of the cone is 12 inches. Find the number of square inches in the area of a section parallel to the base and 8 inches from the vertex. 11.....

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*Directions (12–15):* Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

12 Two face angles of a trihedral angle are  $120^\circ$  and  $150^\circ$ . The third face angle may be (a)  $30^\circ$  (b)  $60^\circ$  (c)  $90^\circ$  12.....

13 Three lines are always coplanar if they (a) pass through the same point (b) are parallel to the same plane (c) are perpendicular to the same line at the same point 13.....

14  $ABCD$  is a spherical quadrilateral on a sphere whose radius is 7. The perimeter of  $ABCD$  may be (a) 54 (b) 44 (c) 34 14.....

15 The locus of points at a given distance  $d$  from a plane and also at the same distance  $d$  from a point on the plane is (a) two points (b) one point (c) one circle 15.....

*Directions (16–20):* For *each* of the following, tell whether the statement is always true, sometimes true or never true by writing the word *always*, *sometimes* or *never* on the line at the right.

16 If a line is parallel to one of two intersecting planes and perpendicular to the other, the two planes are perpendicular. 16.....

17 If two intersecting planes are each tangent to a cylinder, their intersection is parallel to an element of the cylinder. 17.....

18 Planes  $A$  and  $B$  are perpendicular to each other and plane  $P$  is perpendicular to both  $A$  and  $B$ . The intersections of  $P$  with  $A$  and  $B$  form a right angle. 18.....

19 Two vertical trihedral angles are congruent. 19.....

20 A point on a sphere can be the pole of two great circles. 20.....

# FOR TEACHERS ONLY

## SG

### INSTRUCTIONS FOR RATING SOLID GEOMETRY

Wednesday, August 22, 1956— 12 m. to 3 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

#### Part I

Allow  $2\frac{1}{2}$  credits for each correct answer; allow no partial credit. For questions 12–14, allow credit if the pupil has written the correct expression instead of the letter *a*, *b* or *c*.

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|-----------------------|----------------|
| (1) 105               | (12) <i>b</i>  |
| (2) $56\pi$           | (13) <i>c</i>  |
| (3) $3r$              | (14) <i>c</i>  |
| (4) $60\pi$           | (15) <i>a</i>  |
| (5) 80                | (16) always    |
| (6) $\frac{32}{3}\pi$ | (17) always    |
| (7) 5                 | (18) always    |
| (8) 6                 | (19) sometimes |
| (9) $48\pi$           | (20) never     |
| (10) 7.3              |                |
| (11) 24               |                |