The University of the State of New York

THE STATE EDUCATION DEPARTMENT

SPECIAL GEOMETRY (SMSG) EXAMINATION

Monday, June 19, 1967 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. Write your answers in the spaces provided on the separate answer sheet.

1 If the length of the line segment joining the midpoints of two sides of an equilateral triangle is 6, find the perimeter of the original triangle.

2 In rhombus $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E. If $m\angle DAB = 100$, find $m\angle ADE$.

3 Find the length of a diagonal of a cube whose edge is 10 units long.

4 In a circle, a chord of length 30 is drawn perpendicular to a radius. The radius of the circle is 17. What is the distance of the chord from the center of the circle?

5 Write the equation of the circle which has the segment joining points (3,—4) and (—3,4) as a diameter.

6 Given the coordinates of three points, A (1,3), B (0,0), and C (—2,k), find the value of k that will make the points A, B, and C collinear.

7 The medians of $\triangle ABC$ have P as their intersection. If the midpoint of $\overline{BC}$ is D and $AP = 20$, find AD.

8 Chords $\overline{AB}$ and $\overline{CD}$ of a circle intersect at E. If $AE = x + 3$, $EB = 1$, $CE = 2x + 1$, and $ED = 2$, find the value of x.

9 One diagonal of a rhombus is 3 times as long as the other diagonal. If the area of the rhombus is 54 square inches, find the number of inches in the length of the shorter diagonal.

10 The measure of the supplement of angle A is 70 more than 3 times the measure of its complement. Find $m\angle A$.

11 A line intersects sides $\overline{AB}$ and $\overline{AC}$ of $\triangle ABC$ at points D and E, respectively, so that $\overline{DE} \parallel \overline{BC}$. If $AD = 9$, $DB = 6$, and $DE = 12$, find BC.

12 A regular pentagon is inscribed in a circle. Find the measure of the angle formed by a side of the pentagon and a line tangent to the circle at one of the vertices of the pentagon.

13 The ratio of the volumes of two spheres is 1:64. What is the ratio of the diameter of the smaller sphere to the diameter of the larger?

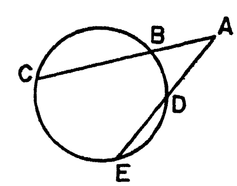
14 Find the length of the longer leg of a right triangle if the altitude to the hypotenuse divides the hypotenuse into two parts whose lengths are 9 and 16, respectively.

15 Two parallel lines, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, intersect transversal $\overleftrightarrow{GH}$ at points G and H, respectively, so that B and D are on the same side of $\overleftrightarrow{GH}$. If $m\angle BGH = 3x - 10$ and $m\angle DHG = 2x + 15$, find $m\angle BGH$.

16 Find the radius of a circle inscribed in a regular hexagon whose side has a measure of 4.

17 The length of an arc of a circle is 8π. If the degree measure of the arc is 120, what is the measure of the radius of the circle?

18 In the accompanying figure, secants $\overleftrightarrow{CB}$ and $\overleftrightarrow{ED}$ intersect at A in the exterior of the circle. If $AB = 6$, $BC = 10$, and $AE = 12$, what is the length of $\overline{ED}$?



19 In right triangle ABC, $\angle C$ is a right angle and $\overline{CD}$ is the median to the hypotenuse. If $m\angle ACD = 36$, what is $m\angle CDB$?

20 Find the area of an isosceles right triangle whose hypotenuse has length 20.

21 If a side of an equilateral triangle has the same measure as the side of a square, what is the ratio of the area of the triangle to the area of the square?

22 In a linear coordinate system, one endpoint of a segment has the coordinate m. If the coordinate of the other endpoint is -3, express the length of the line segment in terms of m.

23 Find the distance between the point whose coordinates are $(-2,5)$ and the point of intersection of the x-axis and the line determined by the equation $y = 2x - 6$.

24 A cross section of area 54 square inches is 9 inches from the vertex of a pyramid whose base has an area of 150 square inches. Find, in inches, the length of the altitude of the pyramid.

Directions (25–34): For each *of those chosen*, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

25 The degree measures of the angles of a triangle are represented as follows: $2x$, $3x + 40$, and $5x - 10$. Which of the following correctly describes the triangle?
(1) acute and scalene
(2) isosceles
(3) obtuse and scalene
(4) right

26 Which of the following conditions would be sufficient to prove two planes parallel to each other?
(1) Two planes intersect a third plane in two parallel lines.
(2) Two planes are parallel to the same line.
(3) Two planes are perpendicular to the same line.
(4) A line in one plane is parallel to the other plane.

27 If two rays are on different faces of a dihedral angle and have their endpoints on its edge, the lines containing these rays
(1) must be parallel
(2) may be parallel
(3) must intersect
(4) may be skew

28 If x is a positive integer and the indirect method of proof is used to prove that x is odd, it would be correct to show that
(1) $x + 1$ is even
(2) "x is even" leads to a contradiction
(3) "x is odd" leads to a contradiction
(4) x^2 is odd

29 Side $\overline{AC}$ of triangle ABC is congruent to side $\overline{ST}$ of triangle RST. If $\angle A \cong \angle S$ and $\angle B \cong \angle R$, the correspondence which can be proven a congruence is
(1) $ABC \leftrightarrow RST$
(2) $ABC \leftrightarrow TRS$
(3) $ABC \leftrightarrow TSR$
(4) $ABC \leftrightarrow SRT$

30 Every line is the edge of
(1) exactly 1 half plane
(2) exactly 2 half planes
(3) infinitely many half planes
(4) exactly 4 half planes

31 If ABC is a triangle not contained in plane E, the projection of $\triangle ABC$ into plane E may be a
(1) point
(2) segment
(3) ray
(4) line

32 If $\angle A$ and $\angle B$ are two angles determined by $\triangle ABC$, the intersection of $\angle A$ and $\angle B$ is
(1) C
(2) AB and C
(3) $\overleftrightarrow{AB}$ and C
(4) $\overline{AB}$ and C

33 The set of points in space equidistant from two points 4 inches apart and 3 inches from one of these points is a
(1) circle
(2) sphere
(3) line
(4) plane

34 Plane E contains $\overline{CD}$, and line L is a perpendicular bisector of $\overline{CD}$. Which of the following must be true?
(1) $\overleftrightarrow{L} \perp E$.
(2) The plane determined by $\overleftrightarrow{L}$ and $\overline{CD} \perp E$.
(3) If point R is on $\overleftrightarrow{L}$, then $RC = RD$.
(4) If $PC = PD$, then P is a point on $\overleftrightarrow{L}$.

35 On the set of coordinate axes *on the answer sheet*, shade in the region which is the intersection of the graphs determined by the equation $x^2 + y^2 \leq 4$ and $x \leq y$.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

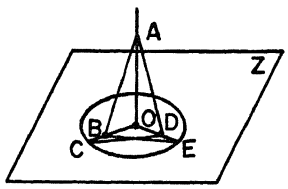
36 Prove *either a* or *b*: [10]

a The sum of the measures of the angles of a triangle is 180.

OR

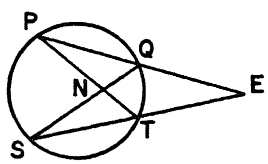
b Given a tangent segment $\overline{QT}$ to a circle, and a secant line through Q, intersecting the circle in points R and S. Then $(QR)\ (QS) = (QT)^2$.

37 Given: Circle O in plane Z, radii $\overline{OC}$ and $\overline{OE}$ of circle O, $\overleftrightarrow{AO} \perp$ plane Z at O, $\overline{AB} \cong \overline{AD}$



Prove: $\overline{CD} \cong \overline{BE}$ [10]

38 In the accompanying figure, $\overrightarrow{PQ}$ and $\overrightarrow{ST}$ intersect at E, $\overline{PT}$ and $\overline{SQ}$ intersect at N, $m\angle PES = 27$, $m\angle PNS = 69$, $m\overset{\frown}{PS} = 24y - 6x$, and $m\overset{\frown}{QT} = 2y + 8x$.



Find:

a the values of x and y [6]

b $m\overset{\frown}{PS}$ [1]

c $m\overset{\frown}{QT}$ [1]

d $m\angle EQS$ [2]

39 Triangle ABC is determined by points A (2,6), B (—3,1), and C (4,0).

a Show, by means of coordinate geometry, that ABC is an isosceles triangle. [3]

b Determine the slope of $\overline{AC}$. [2]

c Write an equation of the line containing the altitude to side $\overline{AC}$. [3]

d Write the coordinates of the point in which the altitude from B intersects $\overline{AC}$. [2]

40 In isosceles trapezoid $ABCD$, base $AB = 16$, base $DC = 10$, and $AD = 6$. [*Answers may be left in radical form.*]

a Find the length of the altitude of the trapezoid. [3]

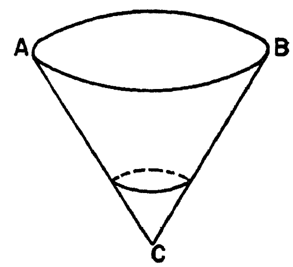
b Find the area of the trapezoid. [2]

c Find the length of diagonal $\overline{DB}$. [2]

d If the bisector of $\angle D$ intersects $\overline{AB}$ at E, find the area of $\triangle AED$. [3]

41 In triangle PQR, $\overline{RT}$ bisects $\angle R$ and T is on $\overline{PQ}$. Prove: $PR > PT$ [10]

42 The diameter of the top of the conical reservoir pictured is 24 feet. The distance from C to B is 20 feet. The reservoir is partially filled with water such that the surface of the water has radius 3 feet.



a Find the number of feet in the height of the reservoir. [2]

b Find, in terms of π, the number of cubic feet in the volume of the reservoir. [2]

c How many feet are there in the vertical distance from the surface of the water to the top of the reservoir? [3]

d Find, in terms of π, the number of cubic feet in the volume of the unfilled portion of the reservoir. [3]

FOR TEACHERS ONLY

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SCORING KEY

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Unless otherwise specified, mathematically correct variations in the answers will be allowed.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.]

(1) 36

(2) 40

(3) $\sqrt{300}$ *or* $10\sqrt{3}$

(4) 8

(5) $x^2 + y^2 = 25$

(6) —6

(7) 30

(8) $\frac{1}{3}$

(9) 6

(10) 80

(11) 20

(12) 36

(13) 1:4

(14) 20

(15) 95

(16) $2\sqrt{3}$

(17) 12

(18) 4

(19) 72

(20) 100

(21) $\sqrt{3}:4$

(22) $|m + 3|$ *or* $|-3 - m|$

(23) $\sqrt{50}$ *or* $5\sqrt{2}$

(24) 15

(25) 1

(26) 3

(27) 4

(28) 2

(29) 4

(30) 3

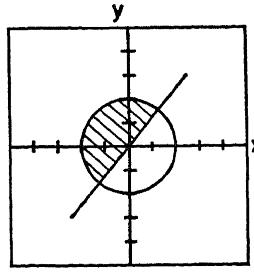
(31) 2

(32) 4

(33) 1

(34) 3

(35)



Part II

(38) *a* $x = 4$, $y = 5$ [6]
b 96 [1]
c 42 [1]
d 132 [2]

(39) *b* —3 [2]
c $y = \frac{1}{3}x + 2$ [3]
d (3,3) [2]

(40) *a* $3\sqrt{3}$ [3]
b $39\sqrt{3}$ [2]
c $\sqrt{196}$ *or* 14 [2]
d $9\sqrt{3}$ [3]

(42) *a* 16 [2]
b 768π [2]
c 12 [3]
d 756π [3]