

SPECIAL GEOMETRY (SMG) EXAMINATION

Tuesday, June 16, 1970—1:15 to 4:15 p.m., only

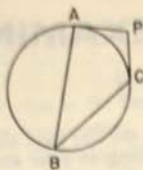
The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.

- Find the number of sides of a regular polygon if one of its exterior angles has a measure of 40.
- Find the area of a rhombus whose diagonals measure 7 and 12.
- In $\triangle XYZ$, $m\angle X = 60$ and $\angle X < \angle Y$. Name the longest side of $\triangle XYZ$.
- In parallelogram $ABCD$, $m\angle A = (2x - 20)$ and $m\angle B = (x + 50)$. Find x .
- Points A and B have coordinates of $(7,3)$ and $(-1,-3)$, respectively. Find AB .
- Find the coordinates of the midpoint of \overline{AB} if the coordinates of A and B are $(3,0,8)$ and $(7,8,4)$.
- In a circle, chords \overline{ST} and \overline{UV} intersect at A . If $SA = (x - 3)$, $AT = (x + 3)$, $AV = (x - 1)$, and $UA = x$, find x .
- A square pyramid has an altitude of measure h . The length of one side of the base is b . Express in terms of b and h the volume of the pyramid.
- Altitude \overline{CD} is drawn to the hypotenuse of right triangle ABC . If $CD = 4$ and $AD = 2$, find DB .
- Find the area of $\triangle ABC$ if $m\angle C = 90$, $m\angle A = 30$, and $BC = 8$.
- Point P has coordinates $(2,5)$. What are the coordinates of the projection of P onto the y -axis?
- Find the lateral surface area of a regular pyramid whose slant height is 5 and whose base is a regular pentagon of side length 2.
- If an equilateral triangle has an inscribed circle with radius 8, what is the radius of its circumscribed circle?
- In $\triangle ABC$, $\angle C$ is a right angle and $m\angle A = 54$. If \overline{CM} is the median to the hypotenuse, find $m\angle BCM$.
- If the radius of circle A is 3 more than the radius of circle B , what is the difference in their circumferences?
- The diagonal of a rectangular parallelepiped has length 26. The lengths of the edges of the parallelepiped are 6, 24, and x . Find x .
- Point P is in the exterior of a circle whose center is A and whose radius is 15. If a tangent segment from P to the circle has length 8, find PA .
- Write an equation of the line which contains the point $(6,2)$ and is parallel to the line whose equation is $3y = 2x - 7$.

- 19 In the accompanying plane figure, \overline{PA} and \overline{PC} are tangent segments and \overline{AB} and \overline{BC} are chords.



If $m\angle B = 40$, find $m\angle P$.

- 20 In rectangle $ABCD$, $AB > AD$. The diagonals intersect at E such that $m\angle CEB = 60$. If $AD = 10$, find AE .

- 21 Three points of a line, A , B , and C have coordinates r , s , and $r + s$, respectively. If $r > 0$ and $s < 0$, which point is between the other two?

- 22 The coordinates of the endpoints of \overline{AB} are $(3,5)$ and $(x,7)$. For what value of x will the slope of \overleftrightarrow{AB} be undefined?

- 23 Find the measure of the angle formed by any two face diagonals of a cube which meet at a vertex.

Directions (24–34): For each of those chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

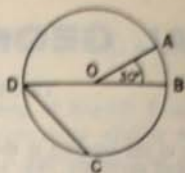
- 24 In $\triangle ABC$, $AB + CB = 2a$ and $AC = 2c$. It follows that

- (1) $a = c$ (3) $c - a > 0$
 (2) $a > c$ (4) $2a < 2c$

- 25 In a linear coordinate system $|x| \leq 2$ is represented by

- (1) one ray (3) a line segment
 (2) two rays (4) two points

- 26 In the accompanying plane figure, \overline{DB} is a diameter, \overline{OA} a radius, and \overline{DC} a chord.



If $m\angle AOB = 30$ and $m\widehat{AB} = \frac{1}{2}m\widehat{BC}$, then $m\angle BDC$ equals

- (1) 5 (3) 45
 (2) 30 (4) 90

- 27 Which is always sufficient to determine a plane?

- (1) a point and a line
 (2) two lines
 (3) three points
 (4) two intersecting lines

- 28 Proving uniqueness and existence of an object is the same as proving that there is (are)

- (1) only one (3) more than one
 (2) at least one (4) one and only one

- 29 Line L is perpendicular to plane E . The set of all points 5 inches from L and 2 inches from plane E consists of

- (1) exactly two circles
 (2) one circle, only
 (3) two parallel lines
 (4) a cylindrical surface

- 30 Parallelogram $ABCD$ is not a rhombus and diagonal \overline{DB} is drawn. Which correspondence is a congruence?

- (1) $ABC \leftrightarrow BDC$ (3) $ABD \leftrightarrow BCD$
 (2) $ABD \leftrightarrow CBD$ (4) $ABD \leftrightarrow CDB$

31 The altitude of a cone is 12 inches. A cross section of the cone is 8 inches from the vertex. What is the ratio of the volume of the smaller cone formed to that of the larger cone?

- (1) 4:9
(2) 2:3
(3) 8:27
(4) 1:27

32 In $\triangle RST$, U is a point between R and S . If area $\triangle RTU = \text{area } \triangle TSU$, then for $\triangle RST$, \overline{TU} must be

- (1) a median
(2) an altitude
(3) an angle bisector
(4) a perpendicular bisector

33 A sphere and a right circular cone have radii of equal measure. Their volumes are also equal. If r represents the radius and h is the height of the cone, the ratio $r:h$ is

- (1) 3:4
(2) 1:4
(3) 4:3
(4) 4:1

34 The line whose equation is $y = x + 2$ is the edge of two half-planes in the coordinate plane. The segment joining the points $(0,3)$ and $(3,0)$

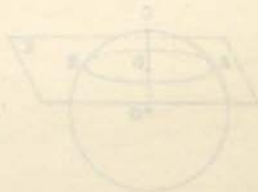
- (1) lies completely in one half-plane
(2) is perpendicular to the edge
(3) coincides with the edge
(4) is parallel to the edge

35 On the set of coordinate axes on the answer sheet, sketch the graph of all (x,y) such that $-2 \leq x \leq 1$.



$$[M] \quad -4(12) = -48 \times 100 = -4800$$

Since the volume of the cone is equal to the volume of the sphere, we can set up the equation $\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$. Simplifying, we get $h = 4r$. Therefore, the ratio $r:h$ is $1:4$.



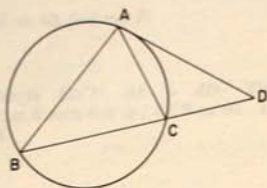
$$[M] \quad 50 \times 100 = 5000$$

Answers to the following questions are to be written on paper provided by the school.

Part II

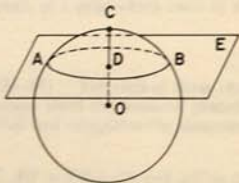
Answer four questions from this part. Show all work unless otherwise directed.

- 36 Given: \overleftrightarrow{AD} tangent to the circle at A and secant \overleftrightarrow{BD} intersecting the circle at B and C



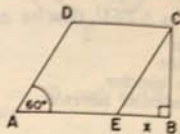
Prove: $BD \times CD = (AD)^2$ [10]

- 37 Points A and B are on the intersection of plane E with a sphere whose center is at O . C is a point on the sphere and $\overline{CO} \perp E$ at point D .



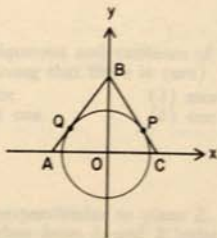
Prove: $\overline{AC} \cong \overline{BC}$ [10]

- 38 $ABCD$ is a trapezoid; $m\angle A = 60$ and $m\angle B = 90$. $\overline{CE} \parallel \overline{AD}$. $CE = AE$ and $EB = x$.



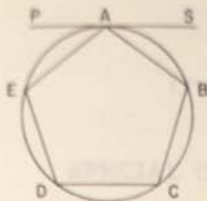
- a Express the area of $\triangle EBC$ in terms of x . [4]
 b Express the area of $AECD$ in terms of x . [3]
 c If $x = 6$, find the area of $ABCD$. [3]

- 39 In the figure points $P(4,3)$ and $Q(-4,3)$ lie on a circle whose center is at the origin O . The tangents drawn at P and Q and the x -axis form $\triangle ABC$.



- a Write an equation of the circle. [3]
 b Find the slope of \overleftrightarrow{OP} . [2]
 c Write an equation of the tangent \overleftrightarrow{BP} . [3]
 d Find OB . [2]

- 40 In the accompanying plane figure $ABCDE$ is an inscribed regular pentagon and \overline{PS} is a tangent segment to the circle at A .



Prove: $\overline{PS} \parallel \overline{CD}$ [10]

- 41 The accompanying diagram represents a capsule composed of a cone fitted with a cylinder so that the base of the cylinder is a cross section of the cone and the vertex of the cone is in the upper base of the cylinder. The radius of the cone is 9 and its height is 15. The height of the cylinder is 5.

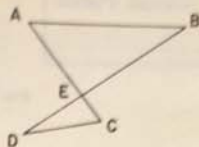


Find the:

- a volume of the cone [2]
 b radius of the cylinder [2]
 c volume of the cylinder [2]
 d volume of the small cone inscribed in the cylinder [2]
 e volume of the capsule [2]

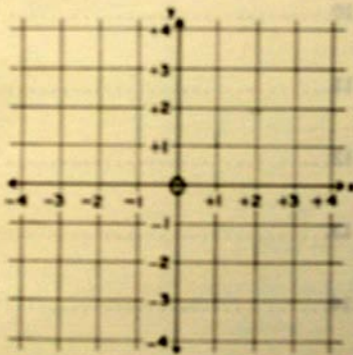
- 42 Choose two questions from a, b, and c.

a In the accompanying plane figure, $m\angle A > m\angle B$ and $m\angle C > m\angle D$.



Prove: $DB > AC$ [5]

- b Three distinct lines, m , n , and p are in one plane with $m \parallel p$ and n intersecting p at a point A . Prove indirectly that n intersects m . [5]
- c In a three-dimensional coordinate system, XYZ , with origin O , the vertices of $\triangle ABC$ are $A(8,0,0)$, $B(0,8,0)$, and $C(8,8,6)$.
- Show that $\triangle ABC$ is isosceles. [3]
 - Write the equation of the plane that contains point C and is parallel to the X - Y plane. [2]



FOR TEACHERS ONLY

The University of the State of New York

THE STATE EDUCATION DEPARTMENT

SCORING KEY

SPECIAL GEOMETRY (SMSG) EXAMINATION

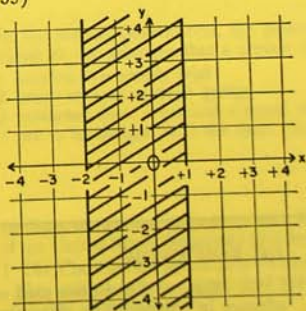
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Unless otherwise specified, mathematically correct variations in the answers will be allowed.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.]

- | | | |
|----------------------------|-----------------------------|--------|
| (1) 9 | (13) 16 | (25) 3 |
| (2) 42 | (14) 36 | (26) 3 |
| (3) \overline{XZ} or y | (15) 6π | (27) 4 |
| (4) 50 | (16) 8 | (28) 4 |
| (5) 10 | (17) 17 | (29) 1 |
| (6) (5,4,6) | (18) $y = \frac{2}{3}x - 2$ | (30) 4 |
| (7) 9 | (19) 100 | (31) 3 |
| (8) $\frac{1}{8}b^2h$ | (20) 10 | (32) 1 |
| (9) 8 | (21) C or $r + s$ | (33) 2 |
| (10) $32\sqrt{3}$ | (22) 3 | (34) 2 |
| (11) (0,5) | (23) 60 | (35) 2 |
| (12) 25 | (24) 2 | |



[OVER]

Part II

- (38) a $\frac{x\sqrt{3}}{2}$ [4] (41) a 405° [2]
 b 3 [2]
 c 45° [2]
 d 15° [2]
 e 435° [2]

- (39) a $x^2 + y^2 = 25$ [3] (42) c (2) $Z = 6$ [2]
 b $\frac{3}{4}$ [2]
 c $4x + 3y = 25$ [3]
 d $2\frac{1}{2}$ [2]

**DO YOU KNOW...**

... that most questions used on Regents examinations have been tried out in advance in representative classrooms throughout the State?

Each year more than 40,000 pupils in about 300 schools "pretest" questions intended for use in future Regents examinations. When committees of classroom teachers meet to assemble Regents examinations, the information obtained from this pretesting is to aid them in determining which questions are appropriate, which questions need revision, and which questions should be eliminated.