TENTH YEAR MATHEMATICS—JANUARY 1956 (1)

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Find the area of a right triangle whose legs are 8 and 10.

2. Find the area of an equilateral triangle whose side is 4.

3. Find the area of a rhombus whose diagonals are 20 and 29.

4. An angle of 64° is inscribed in a circle. Find the number of degrees in its intersected arc.

5. Find the number of degrees in an exterior angle of a regular polygon of eight sides.

6. From a point outside a circle a tangent and a secant are drawn to the circle. The tangent is 8 and the secant is 16. Find the external segment of the secant.

7. In triangle ABC, a line parallel to AB cuts AC at D and BC at E. If $CD = 6$, $AC = 18$ and $BC = 27$, find EC.

8. The areas of two similar triangles are in the ratio 4:9. Find the ratio of a side of the smaller triangle to the corresponding side of the larger triangle.

9. In right triangle ABC, CD is the altitude to the hypotenuse AB. If $CD = 6$ and $AD = 3$, find BD.

10. Find the radius of a circle whose circumference is 16π.

11. The angle of a sector of a circle is 80° and the radius of the circle is 9. Find the area of the sector.

12. A ladder leans against a house standing on level ground. If the foot of the ladder is 4 feet from the house and the top of the ladder is 11 feet from the ground, find to the *nearest degree* the acute angle the ladder makes with the ground.

13. Find the coordinates of the mid-point of the line segment whose end points are $(-2, 0)$ and $(2, 3)$.

14. Find the length of the line segment whose end points are $(4, 0)$ and $(2, 3)$.

15. Write an equation of the straight line through point $(2, 3)$ perpendicular to the x-axis.

Directions (16-19): Indicate the correct completion for *each* statement by writing on the line at the right the letter *a*, *b* or *c*.

16. Line segment AB is 1 inch long. The number of points 2 inches from both A and B is (*a*) 0 (*b*) 1 (*c*) 2

17. Two opposite angles of an isosceles trapezoid are (*a*) equal (*b*) complementary (*c*) supplementary

18. If a base angle of an isosceles triangle is twice the vertex angle, the number of degrees in the vertex angle is (*a*) 36° (*b*) 45° (*c*) 72°

19. The converse of the statement: "Pupils who are G. O. (General Organization) members get a discount at local stores," is

(*a*) pupils get a discount at local stores if they are G. O. members
(*b*) pupils who do not get a discount at local stores are not G. O. members
(*c*) pupils who get a discount at local stores are G. O. members

Directions (20-24): For *each* of the following, tell whether the statement is always true, sometimes true or never true by writing the word *always, sometimes* or *never* on the line at the right.

20. The point of intersection of the perpendicular bisectors of the sides of a triangle lies on a side of the triangle.

21. Triangle ABC is isosceles. If D is any point in base AC between A and C, then BD is greater than AB.

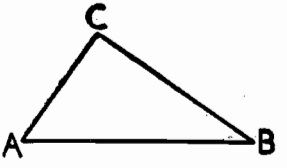
22. If the diagonals of a quadrilateral are equal, the quadrilateral is a rectangle.

23. Two regular polygons are similar if they have the same number of sides.

24. Complementary angles are angles whose sum is a right angle.

Directions (25): Leave all construction lines on your paper.

25. Given triangle ABC. Construct the median to side BC.



Part II

Answer three questions from this part.

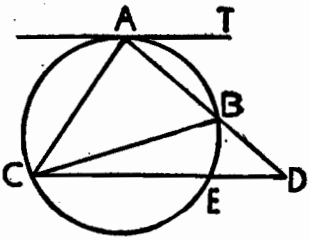
26. Prove: An angle formed by two chords intersecting inside the circle is measured by one half the sum of the intercepted arcs. [10]

27. In isosceles triangle ABC, $AB = BC$. Points D and E are taken on AB and BC respectively so that $BD = BE$. AE and CD are drawn and intersect at H. Prove that

a. $AE = CD$ [6]

b. triangle AHC is isosceles [4]

28. In the diagram at the right AT is tangent to the given circle. CE is a chord parallel to AT, and B is any point on arc AE. AC, BC and AB are drawn. AB and CE are extended to meet at D. Prove that AC is the mean proportional between AB and AD. [10]



29. Prove: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs. [10]

30. Given two concentric circles whose center is O and whose radii are 4 and 8, and given straight line AB through O.

a. Describe *fully* the locus of points equidistant from the two concentric circles. [3]

b. Describe *fully* the locus of points at a given distance d from AB. [4]

c. Find the number of points that satisfy the conditions in both part *a* and part *b* if

(1) $d = 4$ [1]

(2) $d = 6$ [1]

(3) $d = 8$ [1]

*31. The vertices of quadrilateral $RSTV$ are $R(0, 0)$, $S(a, 0)$, $T(a + b, c)$ and $V(b, c)$.

a. Find the slope of RV and the slope of ST. [4]

b. Show that $RSTV$ is a parallelogram. [6]

* This question is based on one of the optional topics in the syllabus and may be substituted for any question in *either* part II or part III.

Part III

Answer two questions from this part. Show all work.

32. A rectangle and a parallelogram have equal areas. The base of the rectangle is 16 and its diagonal is 20. The base of the rectangle and the base of the parallelogram are in the ratio 2:3.

a. Find the altitude of the parallelogram. [8]

b. Indicate the correct completion for the following statement by writing the number (1), (2) or (3) after the letter *b* on your answer paper. [2]

The perimeter of the rectangle is (1) less than the perimeter of the parallelogram (2) equal to the perimeter of the parallelogram (3) greater than the perimeter of the parallelogram

33. In triangle *ABC*, sides *AB*, *BC* and *CA* are represented by $4a - 1$, $3a + 3$ and $4a + 8$, respectively. The perimeter of triangle *ABC* is 54.

a. Find the sides of the triangle. [5]

b. Find the area of the triangle. [5]

34. Tangents *PA* and *PB* are drawn to a circle from external point *P*. Angle $APB = 40°$ and $PA = 25$.

a. Find the number of degrees in minor arc *AB*. [2]

b. Find to the *nearest tenth* the radius of the circle. [4]

c. Find to the *nearest integer* the length of minor arc *AB*. [Use $\pi = \frac{22}{7}$.] [4]

35. The vertices of a triangle are *A*(0, 6), *B*(6, 2) and *C*(12, 10).

a. Find the area of triangle *ABC*. [4]

b. Find the length of *BC*. [2]

c. Find the length of the altitude from *A* to *BC*. [2]

d. Find the equation of the median to *BC*. [2]