The University of the State of New York

10

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Thursday, January 25, 1968 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

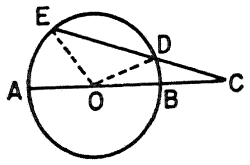
Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 The sides of triangle ABC are represented by $(2x - 1)$, $(x + 9)$, and $(3x - 4)$, respectively. If the perimeter of the triangle is 40, find the length of the *longest side*.

2 Find the coordinates of the midpoint of the line segment joining point A $(-5,-4)$ and point B $(3,-2)$.

3 Find the number of degrees in each of the exterior angles of a regular pentagon.

4 In the accompanying diagram, diameter AB of circle O is extended to C and secant CDE is drawn. If angle $EOD = 110°$ and arc $BD = 20°$, find the number of degrees in angle C.



5 Find the length of a side of a square whose diagonal is $8\sqrt{2}$.

6 The hypotenuse of a right triangle is 10. Find the length of the median to the hypotenuse.

7 Find the length of the line segment joining the points whose coordinates are $(-4,3)$ and $(2,-5)$.

8 In triangle ABC, DE is drawn parallel to AB, intersecting AC in D and BC in E. If DE is one-third AB, what is the ratio of the area of triangle DEC to the area of triangle ABC?

9 A vertical pole on level ground casts a shadow 6 feet long when the angle of elevation of the sun is 67°. Find to the *nearest foot* the height of the pole.

10 If a regular hexagon is inscribed in a circle and a tangent to the circle is drawn at one of the vertices, find the measure in degrees of the acute angle formed by the tangent and a side of the hexagon.

11 Two angles of a triangle are in the ratio of 5:3 and their difference is 40°. Find the number of degrees in the *largest angle* of the triangle.

12 Rectangle $ABCD$ is inscribed in a circle. If $AB = 3$ and $BC = 5$, find in radical form the length of the diameter of the circle.

13 A point on the diameter of a circle divides the diameter into segments of 4 and 9, respectively. A chord is drawn through this point perpendicular to the diameter. Find the length of this *chord*.

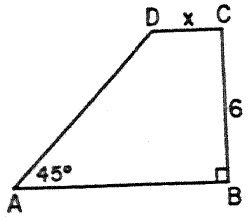
14 In triangle ABC, $AC < AB$ and $AC > BC$. Which angle of the triangle is the largest?

15 If the area of an equilateral triangle is $16\sqrt{3}$, find the length of a side.

16 The length of diameter AB in circle O is 12. Chord AC forms an angle of 30° with AB. Find the distance from the center of the circle to chord AC.

17 In a circle whose radius is 6, the length of an arc is 4π. Find in degrees the measure of this arc.

18 In the accompanying diagram, the altitude of trapezoid $ABCD$ is 6, angle $A = 45°$, and angle $B = 90°$. If base DC is represented by x, express the base AB of the trapezoid in terms of x.



19 If the vertex angle of an isosceles triangle is a right angle, what is the ratio of the altitude drawn to the hypotenuse of the triangle to the hypotenuse?

[OVER]

20 The perimeter of a regular polygon is 50 and the apothem is a. Express the area of the polygon in terms of a.

21 Write an equation of the locus of points for which the abscissa is 3 less than twice the ordinate.

Directions (22–29): For *each* statement or question, write on the separate answer sheet the *number* preceding the word or expression that, of those given, best completes the statement or answers the question.

22 A circle can be inscribed in any
(1) rhombus
(2) parallelogram
(3) trapezoid
(4) rectangle

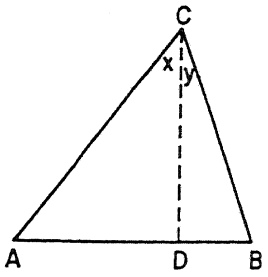
23 It is possible to construct a triangle whose sides are
(1) 7, 7, 16
(2) 5, 7, 12
(3) 4, 8, 13
(4) 5, 6, 10

24 The area of the ring between two concentric circles whose radii are 3 and 4 is
(1) 7
(2) 7π
(3) 25
(4) 25π

25 If two angles of one triangle are equal respectively to two angles of another triangle, these triangles must be
(1) similar
(2) congruent
(3) scalene
(4) isosceles

26 What is the number of points that are equidistant from two parallel lines and also equidistant from two fixed points on one of the lines?
(1) 1
(2) 2
(3) 0
(4) 4

27 In scalene triangle ABC, altitude CD is drawn as shown in the accompanying diagram. If CD divides angle C into two parts, x and y, then



(1) $x + y = A + B$
(2) $x - y = A - B$
(3) $x + A = y + B$
(4) $x + B = y + A$

28 A tangent and a secant are drawn to a circle from an external point. The external segment of the secant is 6 and the internal segment is 5. The length of the tangent is
(1) $\sqrt{30}$
(2) $\sqrt{55}$
(3) $\sqrt{66}$
(4) 6

29 What is the inverse of the statement "If $a = b$, then $2a = 2b$"? [$\neq$ means "does not equal."]
(1) If $2a = 2b$, then $a = b$.
(2) If $a \neq b$, then $2a \neq 2b$.
(3) If $a \neq b$, then $4a^2 \neq 4b^2$.
(4) If $2a \neq 2b$, then $a \neq b$.

30 *On the answer sheet,* construct through point C a line parallel to side AB of triangle ABC.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

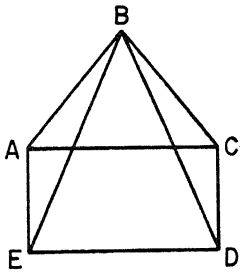
31 Prove *either* *a* or *b* but *not* both: [10]

a An angle formed by two secants is measured by one-half the difference of the intercepted arcs.

OR

b The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

32 In the accompanying plane figure, $AEDC$ is a rectangle and $AB = BC$. If BE and BD are drawn, prove that angle BED = angle BDE. [10]



33 In trapezoid $ABCD$, AB is the longer base. Diagonals AC and BD intersect in E.

a Prove: $\triangle AEB \sim \triangle CED$. [5]

b If the bases of the trapezoid are 5 inches and 15 inches, find the ratio of corresponding altitudes of $\triangle CED$ and $\triangle AEB$. [2]

c If the altitude of the trapezoid is 8 inches, find the number of square inches in the area of $\triangle AEB$. [3]

34 *a* Draw two lines, AB and CD, intersecting at point E and locate a point P on AB but *not* on CD.

(1) *Construct* the locus of points equidistant from AB and CD. [4]

(2) *Construct* the locus of points at a distance EP from E. [2]

(3) Label *all* points which satisfy both conditions in *a* and *b*. [1]

b Describe fully the locus of points in a given plane at a given distance d from a given line m in the plane. [3]

35 The vertices of triangle ABC are A (—1,4), B (5,8), and C (11,0).

a Using graph paper, plot the vertices and draw triangle ABC. [1]

b Find the area of triangle ABC. [4]

c Find the coordinates of D, the midpoint of BC. [2]

d What is the ratio of the measure of AD to the measure of BC? [3]

36 In circle O, sector AOB has a central angle of 40°. The area of the sector is 16π square inches.

a Find the length of the radius of circle O. [2]

b In triangle AOB, find to the *nearest inch* the length of the altitude from A to OB. [3]

c Using the answers obtained in parts *a* and *b*, find to the *nearest square inch* the area of triangle AOB. [2]

d Using the answer obtained in part *c*, find to the *nearest square inch* the area of minor segment AB. [Use the approximation $\pi = 3.14$.] [3]

*37 The vertices of an isosceles triangle ABC are A (0,0), B $(2r,2s)$, and C $(2s,2r)$, where r and s are positive and unequal.

a Find in terms of r and s the coordinates of D, the midpoint of base BC. [2]

b Find the numerical value of the slope of AD. [2]

c Find the numerical value of the slope of BC. [2]

d Write an equation of median AD. [2]

e Point P is a point on the median AD. If its abscissa is k, find its ordinate. [2]

* This question is based on an optional topic in the syllabus.

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Part I Score:.....
Rater's Initials:
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TENTH YEAR MATHEMATICS

Thursday, January 25, 1968 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Pupil..... Teacher.....

School.....

Name and author of textbook used.....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1.....	9.....	17.....
2.....	10.....	18.....
3.....	11.....	19.....
4.....	12.....	20.....
5.....	13.....	21.....
6.....	14.....	22.....
7.....	15.....	23.....
8.....	16.....	24.....

Questions 25 through 30 should be answered on the back of this page.

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[OVER]

25.....

26.....

27.....

28.....

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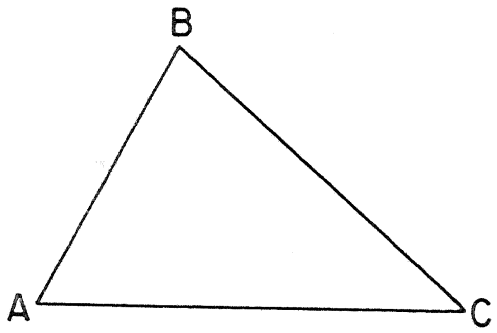
30



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FOR TEACHERS ONLY

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SCORING KEY

TENTH YEAR MATHEMATICS

Thursday, January 25, 1968 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 15

(2) (—1,—3)

(3) 72

(4) 15

(5) 8

(6) 5

(7) 10

(8) 1:9

(9) 14

(10) 30

(11) 100

(12) $\sqrt{34}$

(13) 12

(14) C

(15) 8

(16) 3

(17) 120

(18) $x + 6$

(19) 1:2

(20) $25a$

(21) $x = 2y - 3$

(22) 1

(23) 4

(24) 2

(25) 1

(26) 1

(27) 3

(28) 3

(29) 2

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) *b* 1 : 3 [2]
c 45 [3]

(34) *b* The locus of points in a given plane at a given distance *d* from a given line *m* is two lines parallel to *m*, one on either side of *m* and *d* distance from *m*. [3]

(35) *b* 36 [4]
c (8,4) [2]
d 9 : 10 [3]

(36) *a* 12 [2]
b 8 [3]
c 48 [2]
d 2 [3]

*(37) *a* $(r+s, r+s)$ [2]
b 1 [2]
c —1 [2]
d $y = x$ [2]
e k [2]