TENTH YEAR MATHEMATICS

Monday, June 19, 1950 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in tenth year mathematics, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer two questions from part II.

26 Prove that a diameter perpendicular to a chord of a circle bisects the chord and its minor arc. [10]

27 *ABCD* is a parallelogram with *F* a point on *BC*. A line through *D* and *F* intersects *AB* extended in *E*.

    a Prove: $\dfrac{AE}{DC} = \dfrac{AD}{FC}$ [7]

    b Prove: $AE \times FC = AB \times BC$ [3]

28 In parallelogram *ABCD*, *AD* is longer than *DC* and diagonal *AC* is drawn. Prove that *AC* does not bisect angle *C*. [10]

Part III

Answer two questions from part III.

29 *a* The sides of a triangle inscribed in a circle have arcs represented by $2x - 10°$, $3x + 30°$ and $x + 40°$. Show that two sides of the triangle are equal. [4]

    b A circle whose center is the point (3, 5) passes through the origin. Without constructing the circle, show that the point (8, 2) lies on the circle and that the point (7, 1) does not lie on the circle. [4, 2]

30 In the diagram at the right *AC* is tangent to circle *O* at *C*, *M* is the midpoint of arc *CE* and chord *DE* is parallel to *AC*.

    a If the ratio of arc *CME* to arc *ED* is 5 :8, find the number of degrees in arc *CME* and in arc *ED*. [5]

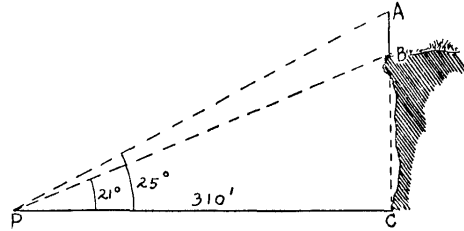   *b* Find the number of degrees in angle *ACE*, angle *DBE* and angle *A*. [1, 2, 2]
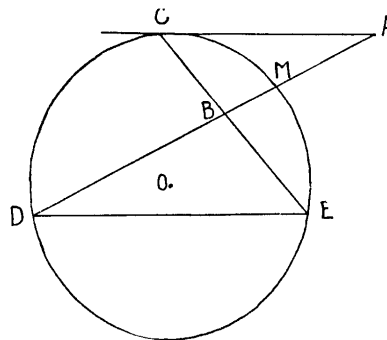
31 In the diagram at the right P represents a point 310 feet from the foot of a vertical cliff BC. AB is a flagpole standing on the edge of the cliff. At P the angle of elevation of B is 21° and of A is 25°.

Find, to the *nearest foot*,

    a the distance AC [4]

    b the length of the flagpole AB [6]

*32 *a* The vertices of parallelogram $ABCD$ have the following coordinates: $A(5,7)$, $B(4,—5)$, $C(—1, 4)$ and $D(0, y)$.

    (1) Find the slope of CB.   [3]

    (2) Express the slope of DA in terms of y.   [2]

    (3) Using the results found in answer to (1) and (2), find the value of y.   [3]

  b Find the abscissa of the point of intersection of the lines $y = 2x + 3$ and $y = —3x + 18$.   [2]

Part IV
Answer one question from part IV.

33 For *each* of the following statements indicate whether the information given is *too little, just enough*, or *more than necessary*, to justify the conclusion.

    a If a polygon is equiangular, then it is regular.   [2]

    b If two line segments join the midpoints of the opposite sides of a quadrilateral, then the line segments bisect each other.   [2]

    c If $x + y = 5$, $x — y = 1$, and $2x — y = 4$, then $x = 3$ and $y = 2$.   [2]

    d If, in every mathematics test, a boy received either an 80% or a 90% grade, then his average mark was 85%.   [2]

    e Mr. A lives in a certain town in which property is assessed at 75% of its true value. If his property is assessed for $6000 and the tax rate is 3%, then the tax on his property can be computed.   [2]

34 A design in the shape of a regular hexagon inscribed in a circle is to be made from a piece of wire 86 inches long. The wire is to be cut into two pieces such that one piece may be used to form the hexagon and the other to form the circle.

    a If x represents the radius of the circle, write an equation which can be used to find x.   [5]

    b Using $\pi = \frac{22}{7}$, find the length of each part of the wire.   [5]

* This question is based upon one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil..Name of school ....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 In a right triangle the altitude on the hypotenuse is 6. One segment of the hypotenuse is 4. Find the other segment.

1................

2 Find the number of degrees in an exterior angle of a regular polygon of 12 sides.

2................

3 In triangle ABC, D and E are the midpoints of AB and BC and DE is drawn. Find the ratio of DE to AC.

3................

4 Two tangents to a circle from an external point are each 6 inches long and they form an angle of 60°. Find the length of the chord joining their points of contact.

4................

5 Two tangents are drawn to a circle from a point outside the circle. One of the intercepted arcs is 100°. Find the number of degrees in the angle formed by the two tangents.

5................

6 In triangle ABC, angle C is a right angle, $AB = 12$ and $AC = 6$. Find the number of degrees in angle B.

6................

7 A tangent and a secant are drawn to a circle from an external point. The secant is 12 and its external segment is 3. Find the length of the tangent.

7................

8 The coordinates of point A are $(a, 2a)$ and of point B $(3a, 4a)$. Find, in terms of a, the coordinates of the midpoint of the line segment AB.

8................

9 Find the length of the line segment AB if the coordinates of point A are $(-3, 0)$ and of point B $(-7, 6)$. [Answer may be left in radical form.]

9................

10 Write the equation of the locus of points equidistant from the points $(4, 10)$ and $(6, 10)$.

10................

11 Corresponding sides of two similar triangles are in the ratio 1:4. Find the ratio of a pair of corresponding altitudes.

11................

12 If two adjacent sides of a parallelogram are 8 and 10 and the included angle is 45°, find the altitude to side 10. [Answer may be left in radical form.]

12................

13 Find the area of an equilateral triangle whose side is 5. [Answer may be left in radical form.]

13................

14 Find the area of a rhombus whose diagonals are 8 and 10.

14................

15 The circumference of a circle is 12π. Find the radius of the circle.

15................

16 Find the area of a sector of a circle whose radius is 5, if the angle of the sector is 40°. [Answer may be left in terms of π.]

16................

17 In isosceles triangle ABC, AB equals BC. Find, to the *nearest integer*, the length of the altitude to AC if angle $ABC = 96°$ and $AB = 10$.

17................

18 If the hypotenuse of a right triangle is 10, find the median to the hypotenuse.

18................

[3]

[OVER]

Directions (questions 19–21) — If the blank in each statement is replaced by one of the words *always, sometimes,* or *never,* the resulting statement is true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

19 If two parallel lines are cut by a transversal, the bisectors of the two interior angles on the same side of the transversal are ... perpendicular to each other.

19...............

20 If diagonal AC of quadrilateral $ABCD$ divides it into two congruent triangles, then the quadrilateral is ... a parallelogram.

20...............

21 If the diagonals of a quadrilateral are unequal and bisect each other at right angles, the quadrilateral is ... a rhombus.

21...............

Directions (questions 22–24) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b* or *c.*

22 The locus of points equidistant from two intersecting lines consists of (*a*)one point (*b*)one line (*c*)two lines

22......

23 If the center of the circle circumscribed about a triangle lies on one side of the triangle, the triangle is (*a*)acute (*b*)right (*c*)obtuse
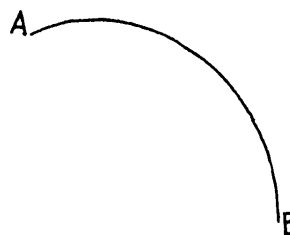
23......

24 To construct a tangent to circle O at point P, a line is drawn perpendicular to OP at point P as shown in the accompanying diagram. Which of the following statements is the theorem used to prove that BP is tangent to circle O?
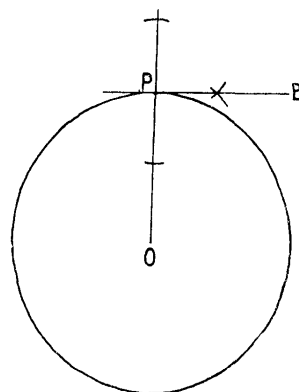
a A tangent to a circle is a line which has one and only one point in common with the circle.

b A line perpendicular to a radius at its extremity on the circle is tangent to the circle.

c A tangent to a circle is perpendicular to the radius drawn to the point of contact.

24......

25 Find by construction the center of the circle of which arc AB is a part.

[4]