

The University of the State of New York
 318TH HIGH SCHOOL EXAMINATION
TENTH YEAR MATHEMATICS
 Monday, June 15, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in tenth year mathematics, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

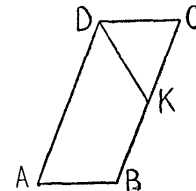
Part II

Answer three questions from part II.

26 Prove: If, in a right triangle, the altitude is drawn upon the hypotenuse, (a) the two triangles thus formed are similar to the given triangle and (b) each leg of the given triangle is the mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to that leg. [5, 5]

27 In parallelogram $ABCD$, side AB is shorter than side BC . K is a point on BC such that DK bisects angle ADC and BK equals AB .

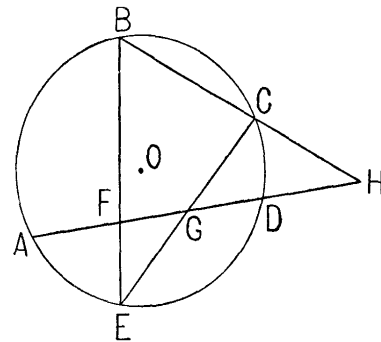
- a Prove: $CK = CD$ [6]
 b Prove that K is the mid-point of BC . [4]



28 Prove: The area of a triangle is equal to one half the product of a side and the altitude drawn to that side. [10]

29 In the figure at the right, HCB and HDA are secants to circle O . Chords EB and EC intersect AH in F and G respectively. Arc AB : Arc BC : Arc $CD = 3:2:1$.

- a Letting n equal the number of degrees in arc CD , find in terms of n the number of degrees in arcs BC and AB . [1]
 b Prove that angle E equals angle H . [4]
 c Prove that triangle GEF is similar to triangle CGH . [3]
 d If CH is 12, EG is 10 and EF is 8, find GH . [2]



30 Prove: Two acute triangles are congruent if two sides and the altitude to one of these sides in one triangle are equal to the corresponding parts of the other triangle. [10]

[1]

[OVER]

Part III

Answer two questions from part III.

31 Acute triangle ABC is inscribed in circle O . The radius of the circle is 10 inches, and sides AB and BC are each 16 inches. OE is drawn perpendicular to side AB , meeting AB at E . OA , OB and OC are drawn.

- a Find to the nearest degree angle EOB . [3]
- b Using the result found in answer to a, find angle AOB and obtuse angle AOC . [1, 3]
- c Using a result found in answer to b, find to the nearest inch the distance from O to chord AC . [3]

32 In trapezoid $ABCD$, base AB is 8 inches and base DC is 20 inches. Sides DA and CB are extended to meet at point G . The altitude of the trapezoid is 3 inches longer than the altitude to side AB of triangle GAB .

- a Using x to represent the altitude from G in triangle GAB , represent the corresponding altitude in triangle GDC . [2]
- b Find the length of the altitude of the trapezoid. [6]
- c Find the area of the trapezoid. [2]

33 a $A(2, 4)$, $B(10, -2)$ and $C(12, 9)$ are the vertices of triangle ABC . Show that triangle ABC is isosceles. [4]

b In parallelogram $ABCD$, diagonals AC and BD meet in E . Side $AD = 25$, $AE = x + 3$, $CE = 3x - 5$, $BE = 3y - 6$ and $DE = 2y + 4$.

- (1) Find x and y . [2]
- (2) Show that parallelogram $ABCD$ is a rhombus. [4]

*34 The vertices of triangle ABC are $A(-4, -3)$, $B(4, 13)$ and $C(6, 7)$.

- a Find the coordinates of M and N , the mid-points of AB and BC respectively. [2]
- b Show by using slopes that MN is parallel to AC . [5]
- c Write the equation of the line passing through M and N . [3]

* This question is based upon one of the optional topics in the syllabus.

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Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The vertex angle of an isosceles triangle contains 70° . Find the number of degrees in a base angle. 1.....

2 Two angles are supplementary, and one is 32° larger than the other. Find the number of degrees in the smaller angle. 2.....

3 Find the number of degrees in an angle of a regular pentagon. 3.....

4 Find the radius of a circle whose area is 36π . 4.....

5 A central angle of a circle whose radius is 18 contains 50° . Find the length of its arc. [Answer may be left in terms of π .] 5.....

6 Find the area of an equilateral triangle whose side is 8. [Answer may be left in radical form.] 6.....

7 The area of a rhombus is 72. If one diagonal is 16, find the other diagonal. 7.....

8 The altitude to the hypotenuse of a right triangle is 12. If one segment of the hypotenuse is 18, find the other segment. 8.....

9 A secant and a tangent are drawn to a circle from an external point. If the secant is 16 and its external segment is 4, find the tangent. 9.....

10 In triangle ABC , angle C equals 90° , angle A equals 71° and AC equals 20 inches. Find to the nearest inch the length of BC . 10.....

11 One side of a polygon is 2, and the corresponding side of a similar polygon is 3. Find the ratio of the areas of the two polygons. 11.....

12 Find the length of the line segment which joins the mid-points of two sides of a triangle whose third side is 10 inches. 12.....

13 A circle whose center is $(7, 2)$ passes through the point $(2, 14)$. Find the radius of the circle. 13.....

14 If the coordinates of A and B are $(-5, 6)$ and $(9, 2)$, find the coordinates of the mid-point of line segment AB . 14.....

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15 All triangles whose vertices are $(2, 2)$, $(6, 2)$ and $(x, 10)$, where x may have any value, are equal in area. Is this statement true or is it false? [Answer true or false.] 15.....

16 Write the equation of the locus of the centers of circles which are tangent to the line $y = 8$ and also tangent to the line $y = 2$. 16.....

Directions (17–22): Indicate the correct completion for each of the following by writing on the line at the right the letter a , b or c .

17 As the number of sides of a polygon increases, the number of degrees in the sum of the exterior angles (a) increases (b) decreases (c) remains the same 17.....

18 A circle can always be circumscribed about any (a) parallelogram (b) rhombus (c) rectangle 18.....

19 Chords AB and CD of a circle intersect in E . If arc AD is $4m^\circ$ and arc BC is $2m^\circ$, then angle AED contains (a) m° (b) $2m^\circ$ (c) $3m^\circ$ 19.....

20 Point C is 3 inches from given line AB . The number of points in AB 5 inches from C is (a) 0 (b) 1 (c) 2 20.....

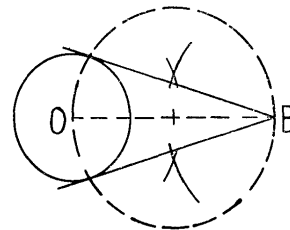
21 If A, B, C and D are four consecutive points on a circle such that arc AB equals arc CD , then chords BC and AD always (a) are equal (b) intersect (c) are parallel 21.....

22 In triangle ABC , angle C equals 60° and AB is greater than AC . Angle B is (a) equal to 60° (b) less than 60° (c) greater than 60° 22.....

- 23 (1) An exterior angle of a triangle is equal to the sum of the nonadjacent interior angles.
 (2) An angle formed by two secants is measured by one half the difference of the intercepted arcs.
 (3) An inscribed angle is measured by one half the intercepted arc.
 (4) A central angle is measured by its intercepted arc.

Which one of the following represents the order in which the above statements should be arranged so that they form a logical sequence? (a) 4, 1, 2, 3 (b) 4, 1, 3, 2 (c) 4, 3, 1, 2 23.....

24 The construction of tangents to circle O from external point B is shown at the right. Which one of the following statements is used to prove the construction? [Answer a or b .]



- a A tangent is perpendicular to a radius drawn to the point of contact.
 b A line perpendicular to a radius at its outer extremity is tangent to the circle. 24.....

Directions (25): Leave all construction lines on your paper.

25 Divide line segment AB into three equal parts. A ----- B