TENTH YEAR MATHEMATICS

Friday, June 17, 1960 — 1:15 to 4:15 p.m., only

Name of pupil...Name of school...

Name and author of textbook used...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 The angles of a triangle are in the ratio of $1:4:5$. Find the number of degrees in the smallest angle of the triangle.　　1................

2 Triangle ABC is inscribed in a circle. If angle $A = 55°$, find the number of degrees in minor arc BC.　　2................

3 Given points A $(-3, 9)$ and B $(11, -5)$. Find the coordinates of the midpoint of the line segment AB.　　3................

4 An interior angle of a regular polygon is $162°$. Find the number of sides of the polygon.　　4................

5 Find the length of the line segment joining the points $(8, 4)$ and $(5, 2)$.　　5................

6 Write an equation of the locus of points such that the sum of the coordinates is 12.　　6................

7 Two tangents are drawn to a circle from a point. If one of the intercepted arcs is $160°$, what is the number of degrees in the angle formed by the two tangents?　　7................

8 In a right triangle the altitude is drawn upon the hypotenuse. If the segments of the hypotenuse cut off by the altitude are 16 and 25, what is the length of the altitude?　　8................

9 The area of a regular polygon is 144 squares inches and its perimeter is 48 inches. Find the number of inches in the length of its apothem.　　9................

10 The radius of one circle is 2 and the radius of a second circle is 5. What is the ratio of the area of the first circle to the area of the second circle?　　10................

11 Find the length of an altitude of an equilateral triangle whose side is 2.　　11................

12 Two points, A and B, are 7 inches apart. How many points are there which are 10 inches from A and 3 inches from B?　　12................

13 The locus of points equidistant from two given concentric circles is a third circle. If the radii of the given circles are 7 and 15, what is the radius of the third circle?　　13................

14 From a point outside a circle a tangent and a secant are drawn to the circle. If the circle divides the secant into an internal segment of 12 inches and an external segment of 4 inches, find the number of inches in the length of the tangent.　　14................

15 In triangle ABC, angle $A = 22°$ and angle $C = 90°$. If side $AC = 5$, find BC to the nearest integer.　　15................

[1]　　　　　　　　　　　　　　　　[OVER]

16 Write an equation of the locus of points equidistant from the points (7, 12) and (7, —6).

16...................

17 If the hypotenuse of a right triangle is $6\sqrt{2}$ and one leg is 6, find the length of the other leg.

17...................

18 Find the area of a rhombus whose diagonals are 5 and 6.

18...................

19 In a circle of radius 9, find the length of an arc of 10°.

19...................

20 Two chords AB and CD of a circle intersect at point P within the circle. If $AP = a$, $PB = b$ and $CP = c$, express the length of PD in terms of a, b and c.

20...................

Directions (21–24): Indicate the correct completion for *each* of the following by writing on the line at the right the *number* 1, 2, 3 or 4.

21 In the accompanying figure, ACB is a straight angle and DC is perpendicular to CE. If the number of degrees in angle ACD is represented by x, the number of degrees in angle BCE is represented by   (1) $90 - x$ (2) $x - 90$ (3) $90 + x$ (4) $180 - x$
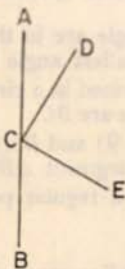
21...................

22 The ratio of the perimeters of two similar pentagons is $4:1$. The ratio of two corresponding sides is   (1) $5:1$ (2) $2:1$ (3) $16:1$ (4) $4:1$

22...................

23 Given: All men are mortal. Which statement expresses a conclusion that logically follows from the given statement?   (1) All mortals are men.   (2) If x is a mortal, then x is a man.   (3) If x is not a mortal, then x is not a man.   (4) If x is not a man, then x is not a mortal.

23...................

24 Given triangle ABC with side AC extended through C to D. If angle BCD is represented by x and angle BCA is represented by y, then for all triangles ABC (1) $\angle x > \angle A$ (2) $\angle x < \angle A$ (3) $\angle x > \angle y$ (4) $\angle x < \angle y$

24...................

Directions (25–28): If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

25 If two triangles have two angles of one equal to two angles of another, the triangles are ... similar.

25...................

26 If a polygon is equilateral, it is ... equiangular.

26...................

27 An inscribed angle which intercepts an arc less than a semicircle is ... an obtuse angle.

27...................

28 The difference between the supplement of an angle and the complement of that angle is ... a right angle.

28...................

Directions (29–30): Leave all construction lines on the paper.

29 Inscribe an equilateral
   triangle in circle O.

O•

A ——————————————————— B
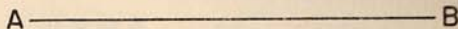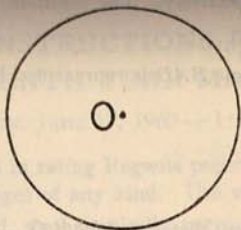
30 Through point P, construct a line parallel to
   line AB.

• P

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either a or b*:

   a The sum of the angles of a triangle is equal to a straight angle.   [10]

   OR

   b If in a right triangle the altitude is drawn upon the hypotenuse,

   (1) the two triangles thus formed are similar to the given triangle and similar to each other [7] and

   (2) each leg of the given triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse.   [3]

32 The vertices of a triangle are A (1, 1), B (3, 5) and C (7, 2).

   a Using graph paper, draw triangle ABC.   [1]

   b Find the area of triangle ABC.   [4]

   c Find the length of BC.   [2]

   d Using the results obtained in *b* and *c*, find the altitude from A to BC.   [3]

[3]

[OVER]

33 Two unequal circles are tangent externally at point A. Line segment BC is a common external tangent touching the circles at B and C, respectively. The common internal tangent at A intersects BC at D.

    a *Prove* that D is the midpoint of BC.   [6]

    b If angle CAD is represented by x and angle BAD is represented by y, *prove* that $x + y = 90°$.   [4]

34 Given point O on line AB.

    a Describe *fully* the locus of points at a given distance d from O.   [2]

    b Describe *fully* the locus of points at a given distance s from AB.   [2]

    c How many points are there which satisfy the conditions given in both a and b if

        (1) $d > s$?   [2]

        (2) $d = s$?   [2]

        (3) $d < s$?   [2]

35 In an isosceles trapezoid $ABCD$, the bases are AB and DC; AD is 5 more than DC and AB is 2 more than twice DC.

    a If DC is represented by x, represent AD and AB in terms of x.   [2]

    b If the perimeter of the trapezoid is 52, find the value of x.   [3]

    c Find the altitude of the trapezoid.   [3]

    d Find the area of the trapezoid.   [2]

36 In rhombus $ABCD$, diagonal $AC = 60$ and angle $BAC = 36°$.

    a Find the length of a side of the rhombus to the *nearest integer*.   [5]

    b Find the length of an altitude of the rhombus to the *nearest integer*.   [5]

*37 Given the points A $(1, 2)$, B $(4, 11)$ and C (x, y).

    a Write an equation of the line through A and C if its slope is —1.   [2]

    b Write an equation of the line through B and C if its slope is $\frac{1}{2}$.   [2]

    c Find the coordinates of point C.   [6]

    * This question is based on optional topics in the syllabus.

[4]

FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING
TENTH YEAR MATHEMATICS

Friday, June 17, 1960 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 21–24, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 18

(2) 110

(3) $(4, 2)$

(4) 20

(5) $\sqrt{13}$ *or* 3.6

(6) $x + y = 12$

(7) 20

(8) 20

(9) 6

(10) $4 : 25$

(11) $\sqrt{3}$ *or* 1.7

(12) one

(13) 11

(14) 8

(15) 2

(16) $y = 3$

(17) 6

(18) 15

(19) $\dfrac{\pi}{2}$ *or* 1.6

(20) $\dfrac{ab}{c}$

(21) 1

(22) 4

(23) 3

(24) 1

(25) always

(26) sometimes

(27) never

(28) always

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10%, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

32 *b* 11   [4]
    c 5   [2]
    d $4\frac{1}{2}$ [3]

34 Phrases such as the following should be allowed credit as indicated:
   a a circle with O as center and d as radius   [2]
   b two lines parallel to AB, one on either side, and each at s distance from AB [2]
   c (1) four   [2]
      (2) two   [2]
      (3) none   [2]

35 *a* $x + 5, 2x + 2$ [2]
    b 8   [3]
    c 12   [3]
    d 156   [2]

36 *a* 37   [5]
    b 35   [5]

37 *a* $y = -x + 3$ [2]
    b $y = \frac{1}{2}x + 9$ [2]
    c $(-4, 7)$ [6]