10

TENTH YEAR MATHEMATICS

Friday, June 17, 1966 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 An exterior angle at the base of an isosceles triangle contains 130°. Find the number of degrees in the vertex angle.

2 One diagonal of a rhombus is twice as long as the other. If the area of the rhombus is 100 square inches, find the number of inches in the length of the shorter diagonal.

3 The number of degrees in a pair of supplementary angles is represented by $(2x - 40)$ and $(3x + 10)$. Find the number of degrees in the smaller of the two angles.

4 In parallelogram $ABCD$, E is the midpoint on side AD and F is the midpoint on side CD. Line EF is drawn. If $EF = 7$, find the length of diagonal AC.

5 Express in radical form the length of the line segment joining the points whose coordinates are $(2,1)$ and $(7,2)$.

6 In the accompanying diagram, circle O is tangent to circle O' at point C. AB is tangent to circle O at A and to circle O' at B. If the length of PC is 4, find the length of AB.
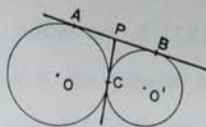
7 If an exterior angle of a regular polygon contains 20°, find the number of sides of the polygon.

8 Find the number of inches in the length of a side of an equilateral triangle whose area is $36\sqrt{3}$ square inches.

9 Two secants, drawn to a circle from the same external point, form an angle of 40°. If the smaller of the two arcs intercepted by the two secants is 30°, find the number of degrees in the larger intercepted arc.

10 What is the total number of points which are equidistant from two intersecting lines m and n and also 3 inches from line m?

11 Side AB of $\triangle ABC$ is 5 inches and side AC is 6 inches. If the number of degrees in angle A varies, what is the largest possible area, in square inches, of $\triangle ABC$?

12 In circle O, a chord AB is 16 inches long and C is the midpoint of this chord. If D is the midpoint of minor arc AB and line segment CD is 4 inches in length, find the number of inches in the length of the diameter of the circle.
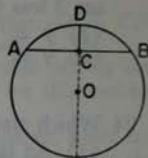
13 The hypotenuse of a right triangle is $2a$ and one leg is a. What is the number of degrees in the smaller acute angle of the triangle?

14 The perimeter of a regular polygon is $8x$ and its apothem is y. Express the area of the polygon in terms of x and y.

15 A tangent and a secant are drawn to a circle from an external point. If the tangent is 12 and the external segment of the secant is 8, find the length of the secant.

16 In right triangle ABC, with the right angle at C, altitude CD is drawn. If $CD = 6$ and the ratio of $AD : DB$ is 1:4, find the length of AD.

17 Chords AB and CD intersect at E within circle O. If $\angle AED = 70°$ and arc $AD = 40°$, find the number of degrees in arc BC.

18 In right triangle ABC, with the right angle at C, AB is 10 and angle A is 25°. Find to the *nearest tenth* the length of BC.

19 The areas of two regular pentagons are in the ratio 4:9. If a side of the smaller pentagon is 10 inches, find the number of inches in the length of a side of the larger pentagon.

20 The area of a circle is 49π. Find the circumference of the circle in terms of π.

21 A sector of a circle has a central angle of $40°$. What is the ratio of the area of the sector to the area of the circle?

Directions (22–28): For *each* statement or question, write on the separate answer sheet the *number* preceding the word or expression that, of those given, best completes the statement or answers the question.

22 If, in triangle ABC, $\angle A = 80°$ and $\angle B = 40°$, then
(1) $BC = 2AC$
(2) AB is the shortest side of $\triangle ABC$
(3) BC is the longest side of $\triangle ABC$
(4) the exterior angle at vertex C is an acute angle

23 An equation of the locus of points whose ordinates are 3 less than twice their abscissas is
(1) $y = 2x - 3$
(2) $y = 2x + 3$
(3) $x = 2y - 3$
(4) $x = 2y + 3$

24 Which statement best illustrates a good *definition*?
(1) An isosceles triangle is a triangle which has two equal sides and two equal angles.
(2) A parallelogram is a polygon whose opposite sides are parallel.
(3) A regular polygon is a polygon which is equilateral and equiangular.
(4) Two adjacent angles are angles which have a common vertex.

25 To prove indirectly that $a > b$, it must be shown that the assumption(s)
(1) $a = b$ will lead to a contradiction of previously given information
(2) $a > b$ and $a = b$ will lead to a contradiction of previously given information
(3) $a < b$ will lead to a contradiction of previously given information
(4) $a < b$ and $a = b$ will lead to a contradiction of previously given information

26 The circle whose center is at the point $(4,1)$ and which passes through the point $(8,1)$ will also pass through the point
(1) $(3,5)$
(2) $(0,1)$
(3) $(4,0)$
(4) $(-4,1)$

27 Which set of numbers may *not* represent the lengths of the sides of a triangle?
(1) 10, 11, 12
(2) 5, 6, 6
(3) 2, 7, 10
(4) 1, 1.1, 1.2

28 The points $(2,-3)$, $(2,3)$ and $(k,0)$ lie on the same straight line. The value of k is
(1) 1
(2) 2
(3) 0
(4) 4

Directions (29–30): Leave all construction lines on the answer sheet.

29 *On the answer sheet*, inscribe an equilateral triangle in circle O.

30 *On the answer sheet*, construct a line through point P which will be parallel to line m.
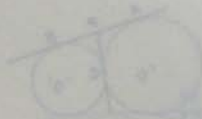
Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either a* or *b* but *not* both: [10]
   a The sum of the angles of a triangle is a straight angle.

<div align="center">OR</div>

   b In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

32 In the accompanying figure, point *M* is the midpoint of minor arc *BC* of circle *O*. A tangent *DME* is drawn to the circle at *M*. Prove that tangent *DME* is parallel to the chord *BC*.   [10]
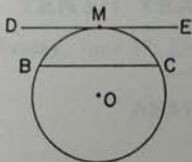
33 In the accompanying figure, *M* is the midpoint of side *BC* in parallelogram *ABCD*. Lines *AB* and *DM* are extended to meet at *E*.
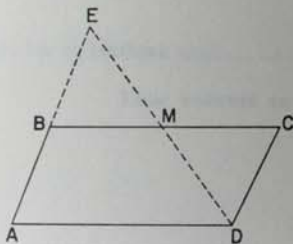
Prove:

a $\triangle BEM \cong \triangle CDM$ [4]
b $AB = BE$ [3]
c the area of parallelogram *ABCD* equal to the area of triangle *ADE*   [3]

34 The vertices of quadrilateral *ABCD* are *A* (1,2), *B* (10,5), *C* (9,8) and *D* (0,5).
   a Find the coordinates of point *E*, the midpoint of *AC*.   [2]
   b Using coordinate geometry, show that the diagonals *AC* and *BD* bisect each other, and write a reason for this conclusion.   [3]
   c Using coordinate geometry, show that the quadrilateral *ABCD* is a rectangle, and write a reason for this conclusion.   [5]

35 The perimeter of isosceles trapezoid *ABCD* is 62. The parallel sides are *AB* and *DC*. Base *AB* is 6 more than four times *DC*, and *AD* is 1 more than twice *DC*.
   a If *DC* is represented by *x*, represent *AB* and *AD* in terms of *x*.   [2]
   b Write in terms of *x* an equation that can be used to find *x*.   [1]
   c Solve for *x* the equation written in answer to *b*.   [1]
   d Find the values of *AB* and *AD*.   [2]
   e Find the area of trapezoid *ABCD*.   [4]

36 In triangle *ABC*, altitude *BD* is drawn to side *AC*. Angle $A = 62°$, angle $C = 20°$ and $BC = 50$ feet.
   a Find the length of *BD* to the *nearest foot*.   [5]
   b Using the result found in answer to *a*, find the length of *AD* to the *nearest foot*.   [5]

*37 The vertices of quadrilateral *ABCD* are *A* (0,0), *B* (6*a*,3*b*), *C* (3*a*,4*b*) and *D* (*a*,3*b*).
   a Using coordinate geometry, show that *AB* ‖ *CD*, and write a reason for this conclusion.   [5]
   b Using coordinate geometry, show that *AD* is *not* parallel to *BC*, and write a reason for this conclusion.   [4]
   c [After the letter *c* on your answer paper, write the *number* of the expression which best completes the statement.]   [1]

   The quadrilateral *ABCD* is a

   (1) rectangle
   (2) rhombus
   (3) trapezoid

   * This question is based on an optional topic in the syllabus.

Math. 10 — June '66

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Friday, June 17, 1966 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Pupil...Teacher...

School...

Name and author of textbook used...

Your answers to part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1............................   9............................   17...........................

2............................   10...........................   18...........................

3............................   11...........................   19...........................

4............................   12...........................   20...........................

5............................   13...........................   21...........................

6............................   14...........................   22...........................

7............................   15...........................   23...........................

8............................   16...........................   24...........................

Questions 25 through 30 should be answered on the back of this page.
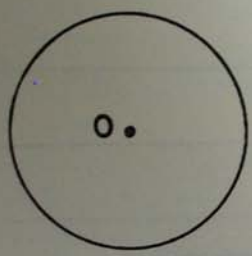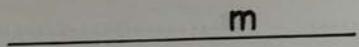
25....................................

26....................................

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29

30

•P

_____ m _____

[8]

Math. 10 — June '66

FOR TEACHERS ONLY

10

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–28, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 80

(2) 10

(3) 44

(4) 14

(5) $\sqrt{26}$

(6) 8

(7) 18

(8) 12

(9) 110

(10) 4

(11) 15

(12) 20

(13) 30

(14) $4xy$

(15) 18

(16) 3

(17) 100

(18) 4.2

(19) 15

(20) 14π

(21) $\frac{1}{8}$

(22) 3

(23) 1

(24) 3

(25) 4

(26) 2

(27) 3

(28) 2

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(34) *a* (5,5)   [2]

  b A statement such as " The midpoint of *BD* coincides with the midpoint of *AC* " would be acceptable.   [3]

  c A statement such as " A parallelogram having one right angle is a rectangle " would be acceptable. [5]

(35) *a* $4x + 6, 2x + 1$ [2]

  b $x + 4x + 2 + 4x + 6 = 62$ [1]

  c 6   [1]

  d 30, 13   [2]

  e 90   [4]

(36) *a* 17   [5]

  b 9   [5]

(37) *a* A statement such as " Lines with the same slope are parallel " would be acceptable.   [5]

  b A statement such as " Lines with different slopes are not parallel " would be acceptable.   [4]

  c 3   [1]