The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 19, 1972 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. On page 5, which is perforated, you will find the "Tables of Natural Trigonometric Functions," which you will need to answer some questions in this examination. Fold this page along the perforation, and tear it off. When you have torn off these two pages and finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 If angle A of parallelogram $ABCD$ measures 42°, find $m\angle B$.

2 In right triangle ABC, $m\angle C = 90$, and the measure of an exterior angle at A is 140°. Which side of the triangle is the *shortest* side?

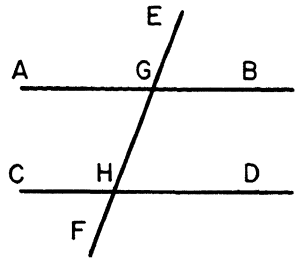
3 The lengths of the two legs of a right triangle are 3 and 4, respectively. Find to the *nearest degree* the measure of the larger acute angle of the triangle.

4 In triangle ABC, $\overline{AD}$ and $\overline{BE}$, the bisectors of $\angle A$ and $\angle B$, respectively, intersect at point F. If $m\angle A = 70$ and $m\angle B = 80$, find $m\angle AFB$.

5 The lengths of the sides of a scalene triangle are 8, 11, and 17, respectively. What is the perimeter of the triangle formed by joining the midpoints of the sides of the original triangle?

6 Two sectors of the same circle have central angles whose measures are 10° and 40°, respectively. What is the ratio of the area of the smaller sector to the area of the larger sector?

7 In the accompanying figure, parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are intersected by transversal $\overleftrightarrow{EF}$ at points G and H, respectively. If $m\angle EGB = (x + 40)$ and $m\angle GHD = (4x - 50)$, find x.



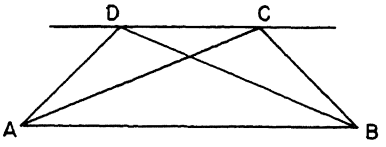
8 A square is circumscribed about a circle whose radius is 2. Find the number of square units in the area of the square.

9 In $\triangle ABC$, $m\angle C = 25$ and $BC = 13$. If the measure of an exterior angle at B is 50°, find AB.

10 The equation $x^2 + y^2 = 16$ represents the locus of all points at a given distance from the origin. What is this distance?

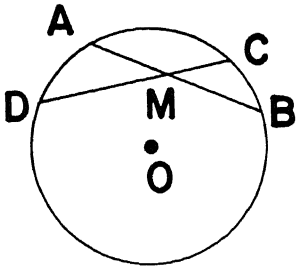
11 The areas of two similar right triangles are 20 and 45, respectively. What is the ratio of the altitude drawn to the hypotenuse of the smaller triangle to the corresponding altitude of the larger triangle?

12 In the accompanying diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$. What is the ratio of the area of triangle ADB to the area of triangle ACB?



13 In right triangle ABC, $m\angle C = 90$ and $m\angle A = 30$. If $AB = 7$, find BC.

14 In the accompanying diagram, chord $\overline{DC}$ bisects chord $\overline{AB}$ at M. If $DM = 8$ and $CM = 6$, find AM. [*Answer may be left in radical form.*]



Directions (15–29): For *each* statement or question, write on the separate answer sheet the *numeral* preceding the word or expression that, of those given, best completes the statement or answers the question.

15 Which is an example of valid reasoning?

(1) All athletes are strong. Jack is an athlete. Therefore Jack is quick.
(2) All athletes are strong. Joe is strong. Therefore Joe is an athlete.
(3) All athletes are strong. Jim is not an athlete. Therefore Jim is not strong.
(4) All athletes are strong. John is an athlete. Therefore John is strong.

16 A triangle and a square may be
(1) collinear
(2) congruent
(3) similar
(4) equal in area

17 If S is the midpoint of $\overline{RT}$, which relationship is *incorrect*?
(1) $RS < RT$
(2) $RS = ST$
(3) $RS > ST$
(4) $RS + ST = RT$

18 Given point P on a line. In a given plane containing the line, the total number of points which are at a distance of 4 units from P and also at a distance of 3 units from the given line is
(1) 1
(2) 2
(3) 3
(4) 4

19 Which statement about parallelograms is true?
(1) A circle may be inscribed in any parallelogram.
(2) The diagonals of any parallelogram are congruent.
(3) The opposite angles of any parallelogram are congruent.
(4) The area of any parallelogram is equal to the product of two adjacent sides.

20 If the length of a side of a square is doubled, then
(1) the perimeter and the area are both doubled
(2) the perimeter and the area are each multiplied by 4
(3) the perimeter is doubled and the area is multiplied by 4
(4) the perimeter is multipled by 4 and the area is doubled

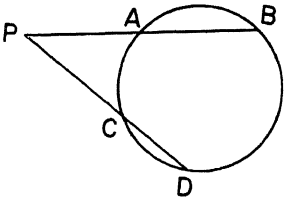
21 If the perimeter of a triangle is 12, which could be the length of one side of the triangle?
(1) 5
(2) 6
(3) 7
(4) 8

22 What are the coordinates of the midpoint of the line segment joining the points whose coordinates are (a,b) and (j,k), respectively?
(1) $(j - a, k - b)$
(2) $\left(\frac{j - a}{2}, \frac{k - b}{2}\right)$
(3) $(j + a, k + b)$
(4) $\left(\frac{j + a}{2}, \frac{k + b}{2}\right)$

23 If four common tangent lines can be drawn to two circles, then the two circles must be
(1) externally tangent
(2) internally tangent
(3) intersecting
(4) completely outside each other

24 If the radius of a circle is 7, then the area of the circle is
(1) exactly 44
(2) exactly 153
(3) between 153 and 154
(4) exactly 154

25 In the accompanying diagram, secants $\overline{PAB}$ and $\overline{PCD}$ are drawn to circle O from external point P. If $\text{m}\overset{\frown}{AB} = 80$, $\text{m}\overset{\frown}{BD} = 144$, and $\text{m}\overset{\frown}{DC} = 62$, then $\text{m}\angle BPD$ equals



(1) 32
(2) 35
(3) 37
(4) 41

26 The area of a regular hexagon inscribed in a circle of radius 4 is
(1) $24\sqrt{3}$
(2) 64
(3) $4\sqrt{3}$
(4) 72

27 The triangle with vertices at (0,0), (5,0), and (—2,—2), respectively, is
(1) right
(2) isosceles but not equilateral
(3) scalene
(4) equilateral

28 A right triangle has hypotenuse of length 16. The length of the projection of one of the legs on the hypotenuse is 9. The length of that leg is
(1) 12
(2) 7
(3) $5\sqrt{7}$
(4) $\sqrt{63}$

29 The sum of the measures of the interior angles of a certain convex polygon is 720°. The sum of the measures of the interior angles of a second convex polygon that has two more sides than the first is
(1) 720°
(2) 900°
(3) 1,080°
(4) 1,440°

Directions (30): Leave all construction lines on the answer sheet.

30 *On the answer sheet,* construct altitude $\overline{DE}$ to side $\overline{AB}$ of given parallelogram $ABCD$.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b*, but *not* both: [10]

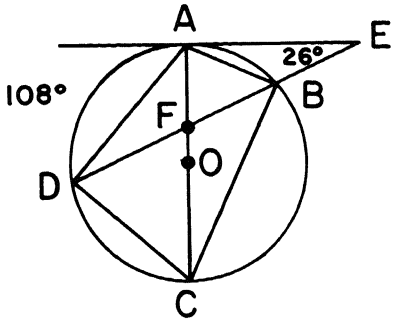
a If two angles of a triangle are equal, the sides opposite these angles are equal.

OR

b The area of a parallelogram is equal to the product of one side and the altitude drawn to that side.

32 Trapezoid $ABCD$ is inscribed in circle O with $\overline{AB} \parallel \overline{DC}$. Prove $\overline{BD} \cong \overline{AC}$. [10]

33 In the accompanying diagram, $\overleftrightarrow{EA}$ is tangent to circle O at A and $\overleftrightarrow{ED}$ intersects circle O at B and D. Diameter $\overline{AC}$ intersects $\overline{BD}$ at F.



If $m\angle E = 26$ and $m\overset{\frown}{AD} = 108$, find:

a $m\overset{\frown}{AB}$ [2]

b $m\overset{\frown}{BC}$ [2]

c $m\angle DFC$ [2]

d $m\angle EAB$ [2]

e $m\angle ABE$ [2]

34 Triangle ABC has vertices $A(-6,-4)$, $B(4,2)$, and $C(0,4)$.

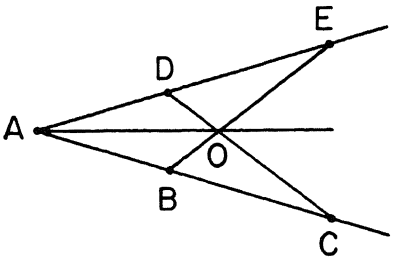
a Using graph paper, draw $\triangle ABC$. [1]

b Find the area of $\triangle ABC$. [5]

c Find the length of $\overline{AC}$. [2]

d Find the length of the line segment joining the midpoints of $\overline{BA}$ and $\overline{BC}$, respectively. [2]

35 Given: $\overline{ADE}$, $\overline{ABC}$, $\overline{AD} \cong \overline{AB}$, $\overline{DE} \cong \overline{BC}$, $\overline{BE}$ and $\overline{DC}$ intersecting at O as shown in the accompanying diagram.



Prove: *a* $\triangle ABE \cong \triangle ADC$ [3]

b $\triangle DOE \cong \triangle BOC$ [3]

c $\overline{AO}$ bisects $\angle DAB$ [4]

36 In right triangle ABC with right angle at C, median $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$. If $AC = 6$ and $BC = 8$, find:

a the length of $\overline{CD}$ [3]

b the area of $\triangle ABC$ [1]

c the length of altitude $\overline{CE}$ to hypotenuse $\overline{AB}$ [3]

d $m\angle ADC$ to the *nearest degree* [3]

*37 The vertices of $\triangle ABC$ are $A(-2,0)$, $B(6,0)$, and $C(8,6)$.

a Write an equation of the line through A parallel to $\overline{BC}$. [3]

b Write an equation of the line through C parallel to $\overline{AB}$. [2]

c The lines whose equations were written in answer to parts *a* and *b* intersect in point D. Find the coordinates of D. [1]

d Show by methods of coordinate geometry that the diagonals of quadrilateral $ABCD$ bisect each other. [4]

*This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY

TENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 15–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 138

(2) $\overline{CB}$

(3) 53

(4) 105

(5) 18

(6) $\frac{1}{4}$

(7) 30

(8) 16

(9) 13

(10) 4

(11) $\frac{2}{3}$

(12) 1

(13) 3.5

(14) $4\sqrt{3}$ or $\sqrt{48}$

(15) 4

(16) 4

(17) 3

(18) 4

(19) 3

(20) 3

(21) 1

(22) 4

(23) 4

(24) 3

(25) 2

(26) 1

(27) 3

(28) 1

(29) 3

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) *a* 56 [2]
b 124 [2]
c 64 [2]
d 28 [2]
e 126 [2]

(34) *b* 22 [5]
c 10 [2]
d 5 [2]

(36) *a* 5 [3]
b 24 [1]
c 4.8 [3]
d 74 [3]

(37) *a* $y = 3x + 6$ [3]
b $y = 6$ [2]
c (0,6) [1]

DO YOU KNOW . . .

Who writes the questions used on Regents examinations?

1 the members of the Board of Regents
2 the subject supervisors in the State Education Department
3 college professors in the various disciplines
4 classroom teachers from schools throughout New York State

The correct answer is 4. Last year more than 400 classroom teachers were involved in the preparation of Regents examination questions, and many other teachers served on the committee that assembled the examinations.