The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 17, 1985 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

On page 9 you will find the "Tables of Natural Trigonometric Functions" which you may need to answer some questions in this examination. Fold this page along the perforations, and tear it off also slowly and carefully.

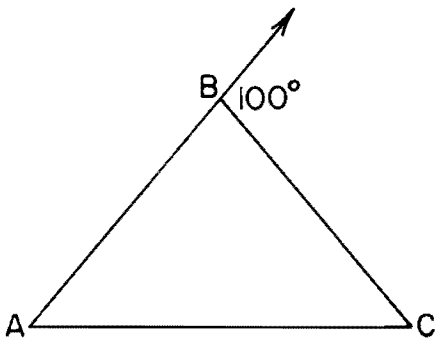
When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

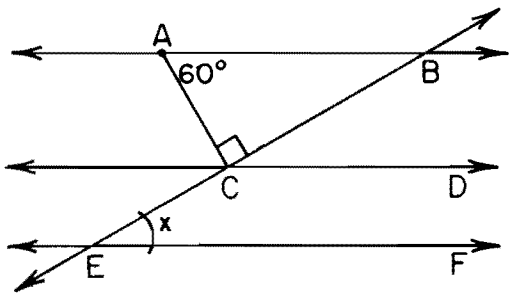
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

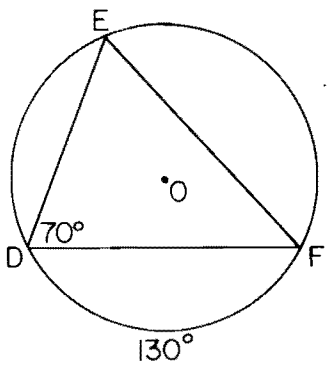
1 In the accompanying diagram of isosceles triangle ABC, $\overline{AB} \cong \overline{BC}$ and the measure of an exterior angle at B is 100°. Find $m\angle A$.



2 In the accompanying figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ with transversal $\overleftrightarrow{BCE}$. If $\angle ACB$ is a right angle and $m\angle BAC = 60$, find $m\angle x$.



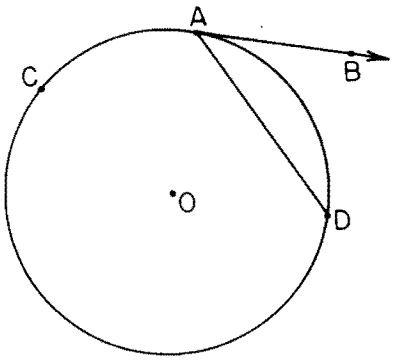
3 In the accompanying diagram, $\triangle DEF$ is inscribed in circle O. If $m\angle D = 70$ and $m\overset{\frown}{DF} = 130$, find $m\angle F$.



4 In circle O, chords $\overline{AB}$ and $\overline{TR}$ are each 5 centimeters from the center O. If $m\overset{\frown}{AB} = 73$, find $m\overset{\frown}{TR}$.

5 The coordinates of the vertices of $\triangle ABC$ are $A(0,6)$, $B(0,0)$, and $C(8,0)$. Find the area of the triangle.

6 In the accompanying diagram, $\overrightarrow{AB}$ is tangent to circle O at A and $m\overset{\frown}{ACD} = 270$. Find $m\angle BAD$.



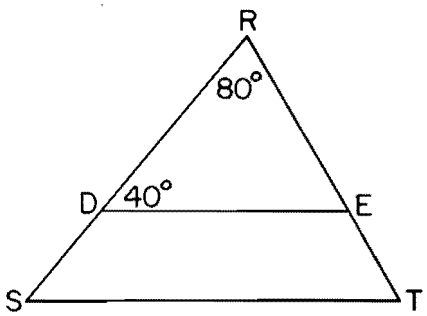
7 In $\triangle RST$, $m\angle R = 50$, $m\angle S = x + 5$, and $m\angle T = x - 5$. Which side of the triangle is the longest?

8 Point M is the midpoint of $\overline{CD}$. The coordinates of C are $(6,-2)$ and the coordinates of M are $(0,0)$. Find the coordinates of point D.

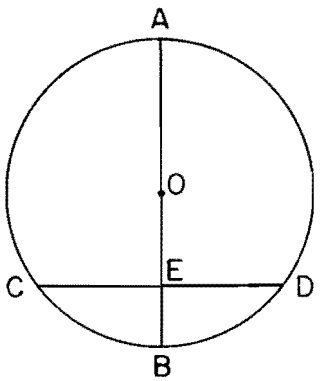
9 The ratio of the areas of two similar triangles is 25:36. What is the ratio of a pair of corresponding sides of these two triangles?

10 What is the slope of the line which passes through the points $(-2,4)$ and $(3,5)$?

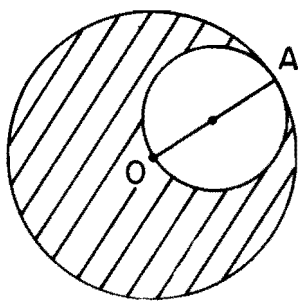
11 In the accompanying diagram of $\triangle RST$, $\overline{DE} \parallel \overline{ST}$, $\overline{RDS}$, $\overline{RET}$, $m\angle R = 80$, and $m\angle RDE = 40$. Find $m\angle T$.



12 In the accompanying diagram of circle O, diameter $\overline{AB}$ is perpendicular to chord $\overline{CD}$ at E. If $CD = 8$ and $OE = 3$, find the length of the radius of the circle.



13 In the accompanying diagram, the radius $\overline{OA}$ of the larger circle is the diameter of the smaller circle. If $OA = 4$, find the area of the shaded region in terms of π.



Directions (14–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

14 Congruent circles may be defined as circles with congruent

(1) centers
(2) chords
(3) central angles
(4) radii

15 Which is an equation of the locus of points 3 units above the x-axis?

(1) $x = 3$
(2) $x = -3$
(3) $y = 3$
(4) $y = -3$

16 In rhombus $ABCD$, the bisectors of $\angle B$ and $\angle C$ *must* be

(1) parallel
(2) oblique
(3) perpendicular
(4) congruent

17 Which is always a regular polygon?

(1) rectangle
(2) square
(3) trapezoid
(4) rhombus

18 The lengths of the sides of a right triangle are 6, 8, and 10. What is the tangent of the *smallest* angle of the triangle?

(1) $\frac{6}{8}$
(2) $\frac{6}{10}$
(3) $\frac{8}{10}$
(4) $\frac{8}{6}$

19 In a triangle whose angles measure 30°, 60°, and 90°, the length of the hypotenuse is represented by $8a$. The length of the side opposite the 60° angle is

(1) $4a$
(2) $4a\sqrt{3}$
(3) $4a\sqrt{2}$
(4) $\sqrt{8a}$

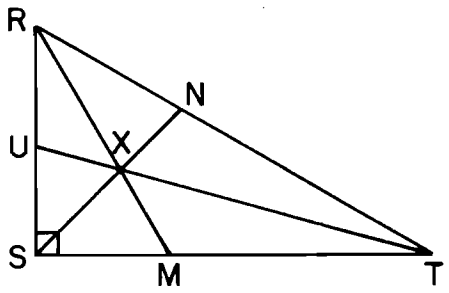
20 In $\triangle ABC$, $m\angle C = 90$ and D is any point on $\overline{AC}$ between A and C. Which statement *must* be true?

(1) $\overline{BD} \perp \overline{AC}$
(2) $CD > BC$
(3) $BC > BD$
(4) $BD > DC$

21 If the area of a trapezoid is 32 and the altitude is 4, the sum of the bases is

(1) 8
(2) 12
(3) 16
(4) 24

22 In right triangle RST, angle bisectors $\overline{RM}$, $\overline{SN}$, and $\overline{TU}$ are drawn and intersect at point X. Which statement is true?



(1) Point X is the center of *only* the inscribed circle.
(2) Point X is the center of *only* the circumscribed circle.
(3) Point X is the center of both the inscribed circle and the circumscribed circle.
(4) Point X is the center of neither the inscribed circle nor the circumscribed circle.

23 The measure of the central angle of a sector of a circle is 120°. If the radius of the circle is 6, the area of the sector is

(1) 4π (3) 6π
(2) 8π (4) 12π

24 The diagonals of parallelogram $ABCD$ intersect at point E. If $DB = 4x + 2$ and $DE = x + 4$, then the value of x is

(1) 1 (3) 3
(2) $\frac{2}{3}$ (4) 0

25 Parallel lines ℓ and m are 6 centimeters apart and P is a point on line ℓ. The total number of points in the plane that are equidistant from ℓ and m and also 3 centimeters from P is

(1) 1 (3) 3
(2) 2 (4) 0

26 If two circles with radii 2 and 5 are drawn such that the distance between their centers is 6, what is the maximum number of common tangents they may have?

(1) 0 (3) 3
(2) 2 (4) 4

27 The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into two segments with lengths 10 and 3. The length of the altitude is

(1) 7 (3) $\sqrt{109}$
(2) 13 (4) $\sqrt{30}$

28 The lines whose equations are $x = 3$ and $y = 2x - 1$ intersect at the point whose coordinates are

(1) $(3,-1)$ (3) $(3,5)$
(2) $(2,3)$ (4) $(0,-1)$

29 Given the statement: "If two triangles are congruent, then they are equal in area." Which of the following is true?

(1) The statement is true but its inverse is false.
(2) The statement is true and its inverse is true.
(3) The statement is false and its inverse is false.
(4) The statement is false but its inverse is true.

30 Which equation can *not* be derived from the proportion $\frac{a}{b} = \frac{r}{s}$?

(1) $\frac{a}{r} = \frac{b}{s}$ (3) $\frac{a+b}{b} = \frac{r+s}{s}$
(2) $\frac{a}{s} = \frac{b}{r}$ (4) $\frac{b}{a} = \frac{s}{r}$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

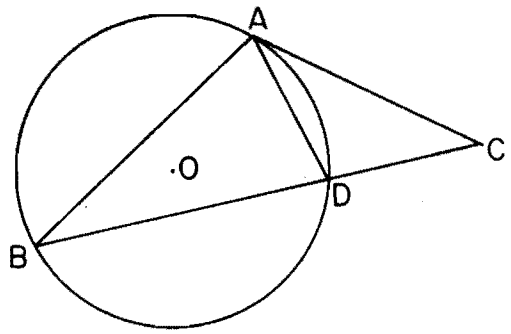
31 Prove either *a* or *b* but *not* both.

a The measure of an angle formed by two secants is equal to one-half the difference of the measures of the intercepted arcs. [10]

OR

b The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs. [10]

32 Given: tangent $\overline{CA}$ and secant $\overline{CDB}$ are drawn to circle O, chord $\overline{AB} \cong$ chord $\overline{BD}$, and $m\angle CAD = 30$.



Find: *a* $m\overset{\frown}{AD}$ [2]

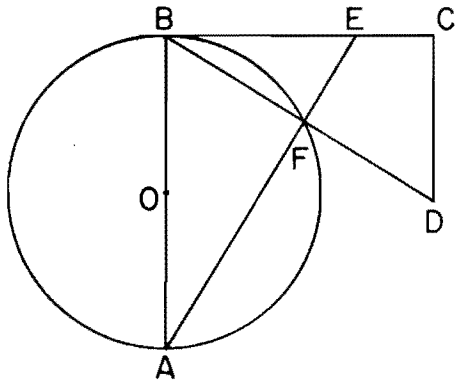
b $m\angle B$ [2]

c $m\overset{\frown}{BD}$ [2]

d $m\angle BAC$ [2]

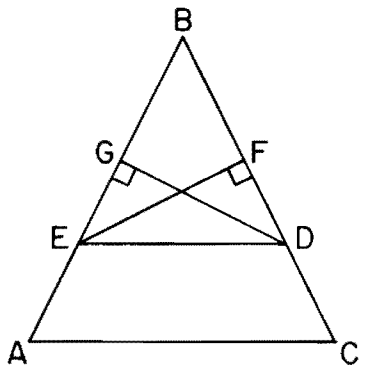
e $m\angle C$ [2]

33 Given: $\overline{CEB}$ is tangent to circle O at B, diameter $\overline{BA} \parallel \overline{CD}$. Secants $\overline{DFB}$ and $\overline{EFA}$ intersect circle O at F.



Prove: $\frac{BD}{AE} = \frac{CD}{BE}$ [10]

34 Given: $\triangle ABC$, $\overline{AB} \cong \overline{BC}$, $\overline{ED} \parallel \overline{AC}$, $\overline{DG} \perp \overline{AB}$, $\overline{EF} \perp \overline{BC}$, $\overline{BGEA}$, and $\overline{BFDC}$.



Prove: $\overline{GE} \cong \overline{FD}$ [10]

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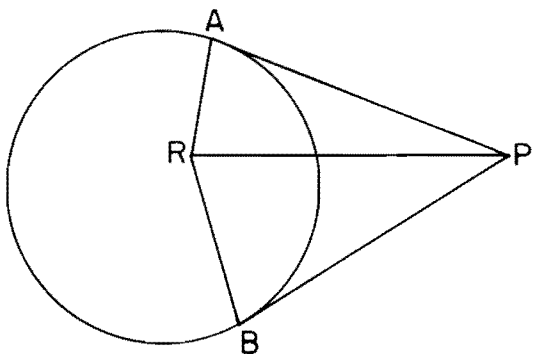
35 Given: $\overline{PA}$ and $\overline{PB}$ are tangent to the circle at A and B, respectively, R lies in the interior of the circle, $\overline{AR}$ and $\overline{BR}$ are drawn, and $\overline{RP}$ does *not* bisect $\angle APB$.



Prove: R is *not* the center of the circle. [10]

36 A circle is inscribed in a regular polygon of 10 sides. The radius of the circle is 12.

a Find the length of the apothem of the polygon. [2]

b Find the length of a side of the polygon to the *nearest tenth*. [5]

c Using the results from parts *a* and *b*, find the area of the polygon to the *nearest integer*. [3]

*37 Given: $\triangle ABC$ with vertices $A(0,0)$, $B(8,3)$, and $C(0,6)$.

a Show that $\triangle ABC$ is an isosceles triangle. [3]

b Find the area of $\triangle ABC$. [3]

c Write an equation of the line through C perpendicular to $\overline{AB}$. [4]

*This question is based on an optional topic in the syllabus.