The University of the State of New York

10

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Wednesday, August 16, 1967 — 8:30 to 11:30 a.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

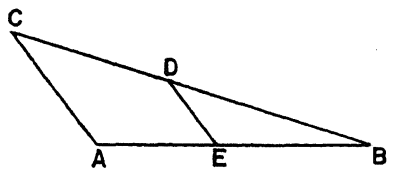
Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 The angles of a triangle are in the ratio 5 : 6 : 7. Find the number of degrees in the smallest angle of this triangle.

2 Find the area of a rhombus whose diagonals are 6 and 11.

3 How many degrees are there in each interior angle of a regular polygon which has 18 sides?

4 In the accompanying figure, ABC is a triangle and DE is parallel to CA.



If $AE = 6$, $EB = 8$, and $DE = 4$, find the length of CA.

5 Find the length of the line segment joining the points whose coordinates are $(-2,-7)$ and $(6,8)$.

6 In triangle ABC, $AB = AC$ and the measure of angle A is twice the measure of angle B. Find the number of degrees in the measure of the exterior angle at C.

7 Express in terms of x the number of degrees in the supplement of an angle of $(80 - 3x)$ degrees.

8 The area of a circle is 16π. Express the circumference of the circle in terms of π.

9 Two similar triangles have corresponding medians in the ratio 2 : 3. What is the ratio of the area of the smaller triangle to the area of the larger triangle?

10 A regular hexagon is inscribed in a circle with a radius of 10. Find the perimeter of the hexagon.

11 If the area of a certain trapezoid is 64 and its bases are 20 and 12, find the length of the altitude.

12 When the angle of elevation of the sun is 50°, a vertical pole on level ground casts a shadow 11 feet long. Find to the *nearest foot* the height of the pole.

13 A chord 12 inches long is drawn in a circle whose radius is 10 inches. Find in inches the distance from the center of the circle to the chord.

14 A line segment has one endpoint at $(3,-2)$, and its midpoint at $(2,-5)$. What are the coordinates of the other endpoint of the line segment?

15 In a circle, an angle formed by a tangent and a chord intercepts the same arc as a central angle. What is the ratio of the measure of the angle formed by the tangent and the chord to the measure of the central angle?

16 In triangle ABC, angle C is a right angle and CD is the altitude drawn upon side AB. If $AD = 2$ and $DB = 18$, find the length of CD.

17 If the bases of a trapezoid are 27 and 11, find the length of the median.

18 A sector of a circle has an area of 12π. If the radius of the circle is 6, find the number of degrees in the central angle of the sector.

19 An angle formed by a tangent and a secant drawn to a circle from the same external point intercepts arcs whose measures are 140° and 30°. Find the number of degrees in the measure of this angle.

20 Express in radical form the length of a diagonal of a rectangle whose adjacent sides are 6 and 3.

21 Write an equation of the locus of points equidistant from the points whose coordinates are (1,3) and (1,7).

Directions (22–29): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes the statement or answers the question.

22 An acute triangle can *not* be
(1) scalene
(2) equilateral
(3) right
(4) isosceles

23 If the lengths of two sides of a triangle are 7 and 8, respectively, the length of the third side may be
(1) 1
(2) $\frac{1}{2}$
(3) 5
(4) 15

24 The radius of the circle inscribed in an equilateral triangle is 5. The length of an altitude of this triangle is
(1) 10
(2) 15
(3) $5\sqrt{3}$
(4) $10\sqrt{3}$

25 If in triangle ABC angle $B = 60°$ and $AB > AC$, then which relationship must be true?
(1) angle $C >$ angle A
(2) angle $C <$ angle A
(3) angle $C =$ angle A
(4) angle $C <$ angle B

26 "One and only one straight line can be drawn through two given points." This sentence can best be classified as
(1) a theorem
(2) a statement assumed to be true
(3) a definition
(4) an undefined term

27 The locus of points 2 inches from a given line and 3 inches from a point on that line is exactly
(1) 1 point
(2) 2 points
(3) 3 points
(4) 4 points

28 Given the statements:
a All quadrilaterals are polygons.
b All parallelograms are quadrilaterals.
c All rectangles are parallelograms.

Based on the above statements, in which sequen should the terms quadrilateral, polygon, parallelogram, and rectangle be defined?
(1) quadrilateral, polygon, parallelogram, rectangle
(2) quadrilateral, polygon, rectangle, parallelogram
(3) polygon, quadrilateral, parallelogram, rectangle
(4) polygon, quadrilateral, rectangle, parallelogram

29 A tangent and a secant are drawn to a circle from an external point. If the length of the tangent is x, the length of the internal segment of the secant is m, and the length of the external segment of the secant is n, then which equation is correct?
(1) $x = n(m + n)$
(2) $nx = m + n$
(3) $x^2 = mn$
(4) $x^2 = n(m + n)$

30 *On the answer sheet,* construct through point B a line perpendicular to line BC.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

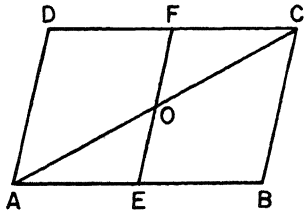
31 Prove *either* *a* or *b* but *not* both: [10]

a Two right triangles are congruent if the hypotenuse and a leg of one are equal to the corresponding parts of the other.

OR

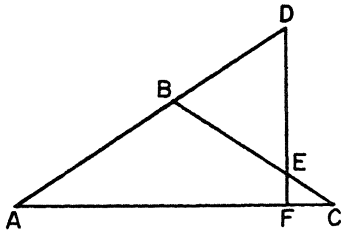
b The area of a triangle is equal to one-half the product of a side and the altitude drawn to that side.

32 In the accompanying figure, $ABCD$ is a quadrilateral. The midpoints of AB and CD are E and F, respectively. Diagonal AC and line segment EF bisect each other at O.



Prove: $ABCD$ is a parallelogram. [10]

33 In triangle ABC, $AB = BC$. Side AB is extended through B to D and DF is drawn perpendicular to AC.



If DF and BC intersect at point E, prove $BD = BE$. [10]

34 In trapezoid $ABCD$ whose bases are AB and DC, $\angle A = (2x)°$, $\angle B = (3x - 10)°$, $\angle C = (2y)°$, and $\angle D = (2y + 20)°$.

a In terms of x and y, write a set of equations that can be used to solve for x and y. [2, 2]

b Solve for x and y the set of equations written in answer to *a*. [4]

c Find the number of degrees in $\angle B$ and $\angle D$. [1, 1]

35 The vertices of triangle ABC are A (—5,10), B (—10,—2), and C (2,—7).

a Using formulas of coordinate geometry,

(1) show that triangle ABC is a right isosceles triangle and state reasons for your conclusion [6]

(2) find the coordinates of the center of the circle which could be circumscribed about $\triangle ABC$ [2]

b Find the area of triangle ABC. [2]

36 A regular pentagon is inscribed in a circle whose radius is 15. Find

a the apothem to the *nearest tenth* [5]

b a side to the *nearest tenth* [3]

c the area (of the polygon) to the *nearest integer* [2]

*37 The bases of trapezoid $ABCD$ are AB and DC. The coordinates of A are (—2,0) and of B are (2,—2).

a Write an equation of the line through A and B. [4]

b The line through D and C intersects the y-axis at a point two units above the origin. Write an equation of this line. [4]

c If an equation of the line through A and D is $x = -2$, find the coordinates of vertex D. [2]

*This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

10

SCORING KEY
TENTH YEAR MATHEMATICS

Wednesday, August 16, 1967 — 8:30 to 11:30 a.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 50

(2) 33

(3) 160

(4) 7

(5) 17

(6) 135

(7) $100 + 3x$

(8) 8π

(9) 4:9

(10) 60

(11) 4

(12) 13

(13) 8

(14) (1,—8)

(15) 1:2

(16) 6

(17) 19

(18) 120

(19) 55

(20) $\sqrt{45}$ *or* $3\sqrt{5}$

(21) $y = 5$

(22) 3

(23) 3

(24) 2

(25) 1

(26) 2

(27) 4

(28) 3

(29) 4

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(34) *a* $(2x)° + (2y + 20)° = 180°$
$(3x - 10)° + (2y)° = 180°$ [2,2]

b $x = 30°$ and $y = 50°$ [4]

c $\angle B = 80°$ and $\angle D = 120°$ [1,1]

(35) *a* (2) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ [2]

b $84\frac{1}{2}$ [2]

(36) *a* 12.1 [5]

b 17.6 [3]

c 532 [2]

(37) *a* $x + 2y + 2 = 0$ [4]

b $x + 2y - 4 = 0$ [4]

c (—2,3) [2]