TRIGONOMETRY

Thursday, January 29, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 Solve the following equation for all positive values of A less than $360°$: [10]
$$\cos 2A + \sin A = 0$$

22 a By substituting $-B$ for B in the formula for $\cos (A + B)$, derive the formula for $\cos (A - B)$. [3]

b Derive the law of sines for the acute plane triangle. [7]

23 In triangle ABC, angle $C = 90°$. If angle DAC is x and angle ABC is $2x$, prove that $d = 2c \sin x$. [10]
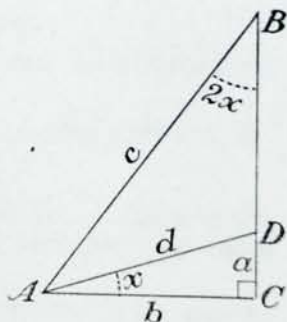
24 a Draw the graph of $y = \cos \frac{1}{2}x$ as x varies from 0 to 2π at intervals of $\frac{\pi}{3}$. [4]

b On the set of axes used in a, draw the graph of $y = 2 \cos x$ as x varies from 0 to 2π at intervals of $\frac{\pi}{3}$. [4]

c From the graphs made in answer to a and b, state the *number* of values of x between 0 and 2π for which $\cos \frac{1}{2}x = 2 \cos x$. [2]

[1]

Part III

Answer at least one question from part III.

25 In triangle ABC, $a = 32.5$, $b = 57.3$ and $c = 61.2$. Find angle B to the *nearest minute*. [10]

26 Two straight roads meet at P, making an angle of $82° 20'$. Camp A located on one road is 238 rods from P. Camp B on the other road is 176 rods from P. A direct road from A to B is to be constructed. At what angle with AP should this road be started? [Find the angle to the *nearest minute*.] [10]

27 A ship is sailing on course N 40° E. From point A on its course, the bearing of lighthouse C is found to be N 75° E. When the ship has steamed 5.8 miles to point B, the bearing of the lighthouse from B is S 62° E. Find the distance BC to the *nearest tenth of a mile*. [10]

Part IV

Answer at least one question from part IV.

28 In spherical triangle ABC, $a = 158°$, $b = 99° 40'$ and $c = 90°$. Find angle C to the *nearest minute*. [10]

29 An aviator flew along the great circle route from Paris (Lat. 48° 50′ N, Long. 2° 20′ E) to Osaka (Lat. 34° 40′ N, Long. 135° 30′ E). How many nautical miles did he fly? [10]

TRIGONOMETRY

Fill in the following lines:

Name of pupil................................Name of school................................

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express cos 290° as a function of a positive acute angle.   1....................

2 Express 105° in radian measure. [Answer may be left in terms of π.]   2....................

3 Find the logarithm of 3.142   3....................

4 Find log sin 63° 48′   4....................

5 Find to the *nearest minute* the positive acute angle whose cosine is 0.8330   5....................

6 In triangle ABC, $A = 30°$, $a = 6.5$ and $b = 13$. Find B.   6....................

7 In triangle ABC, $a = 8$, $b = 13$ and $c = 15$. Find B.   7....................

8 In triangle ABC, $B = 105°$ and $C = 15°$. Find the value of $(b + c) : (b - c)$. [Answer may be left in radical form.]   8....................

9 If A is a positive acute angle, express cot A in terms of sin A.   9....................

10 If A is an angle in the second quadrant and $\sin A = \frac{5}{13}$, find sin $2A$.   10....................

11 If x is an acute angle and $\cos x = \frac{1}{8}$, find $\cos \frac{x}{2}$ 11....................

12 Express sin $3A$ + sin A as a product of two functions.   12....................

Directions (questions 13–17) — Indicate the correct answer to *each* question by writing the letter a, b or c on the line at the right.

13 As an angle increases from π to 2π, its cosine   (a) increases throughout   (b) decreases and then increases   (c) increases and then decreases   13......

14 The value of sin² $2A$ + cos² $2A$ is   (a) 1   (b) 2   (c) 4   14......

15 From the data $B = 35°$, $b = 6$ and $a = 10$, there can be constructed   (a) no triangle   (b) only one triangle   (c) two triangles   15......

16 If in spherical triangle ABC, $C = 90°$, $c = 70°$ and $b = 50°$, then a is   (a) greater than 90°   (b) equal to 90°   (c) less than 90°   16......

17 The statement cos $2x$ = (cos x − sin x) (cos x + sin x) is   (a) true for all values of x (b) true for only certain values of x (c) not true for any value of x 17......

18 Find to the *nearest integer*, the area of triangle ABC in which $a = 12$, $c = 15$ and $B = 18°$.   18......

19 Find the value of x greater than 0° and less than 90° which satisfies the equation sin $2x$ = cos x.   19......

20 In right spherical triangle ABC in which C is the right angle, a and b are known. Write a formula that can be used to find A.   20......

[3]