TRIGONOMETRY

Wednesday, January 27, 1954 — 9.15 a. m. to 12.15 p. m., only

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year,

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 *a* Prove the identity: $\dfrac{\sin \theta}{1 + \cos \theta} + \dfrac{1 + \cos \theta}{\sin \theta} = 2 \cot \theta \sec \theta$ [7]

   b Starting with the formula for $\sin (x + y)$, derive the formula for $\sin 2x$.   [3]

22 Find to the *nearest degree* all values of θ greater than 0° and less than 360° that satisfy the following equation: $3 \cos 2\theta + 8 \sin \theta + 5 = 0$ [10]

23 *a* Sketch the graph of $y = 2 \sin x$ as x varies from 0 to 2π radians.   [4]

   b On the same set of axes used in *a*, sketch the graph of $y = \cos \frac{1}{2} x$ as x varies from 0 to 2π radians.   [4]

   c From the graphs made in answer to *a* and *b*, determine the number of values of x between 0 and 2π radians for which $2 \sin x = \cos \frac{1}{2} x$.   [2]

24 In the figure at the right BC is perpendicular to AC, $AD = b$, $BC = h$, angle $ABD = 30°$ and angle $BDC = x$.
Prove: $h = b \sin x (\sqrt{3} \sin x - \cos x)$ [10]

[1]
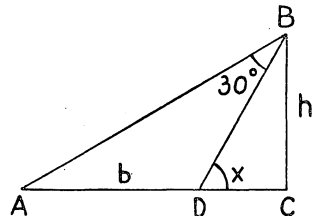
Part III
Answer at least two questions from part III.

25 Engineers wish to build a bridge across a stream to join point C on one side with either point A or point B on the other side. The distance from A to B is 325 feet, angle ABC is 79° 30′ and angle BAC is 72° 20′. By how many feet does the distance from A to C exceed the distance from B to C? [5, 5]

26 Two forces of 178 pounds and 226 pounds act on a body and produce a resultant of 312 pounds. Find to the *nearest degree* the angle formed by the lines of action of the resultant and the larger force. [10]

27 A building and a flagpole are on level ground as shown in the figure at the right. From B, a point on the ground at the base of the building, the angle of elevation of A, the top of the pole, is 72°. From a window (C) 63 feet directly above B the angle of elevation of A is 38°. Find to the *nearest foot* the height of pole AD. [5, 5]
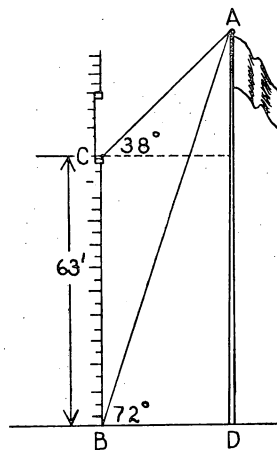
28 Airfield B is 74 miles N 14° 20′ E from airfield A. An airplane leaves field B and flies in the direction N 85° 40′ E to C where it is forced to make an emergency landing. If C is 105 miles from B, find to the *nearest degree* angle BAC. [5, 5]

|2|

Fill in the following lines:

Name of pupil.. Name of school...

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express cos 290° as a function of a positive acute angle.   1................

2 In a circle whose radius is 2 inches, find the length of the arc intercepted by a central angle of 1.25 radians.   2................

3 Express in radians an angle of 198°. [Answer may be left in terms of π.]   3................

4 Find the smallest positive value of x for which $\cot x = 3 \tan x$.   4................

5 If A is a positive acute angle, express $\cot A$ in terms of $\sin A$.   5................

6 The area of an acute triangle is 18. If $a = 8$ and $b = 9$, find C.   6................

7 Find the logarithm of 58.06   7................

8 Find cos 34° 34′   8................

9 Find to the *nearest minute* the positive acute angle A for which $\log \tan A = 0.5828$ 9................

10 Two forces of 12 pounds and 5 pounds act upon a body at right angles to each other. Find to the *nearest degree* the angle formed by the lines of action of the resultant and the larger force.   10................

11 In triangle ABC, $\sin A = 0.3$, $\sin B = 0.5$ and $b = 20$. Find a.   11................

12 In triangle ABC, $a = 10$, $b = 6$ and $C = 100°$. Find to the *nearest hundredth* $\tan \frac{1}{2}(A - B)$.   12................

13 In triangle ABC, $a = 5$, $b = 9$ and $\cos C = \frac{7}{15}$. Find c.   13................

[3]   [OVER]

14 If A is an acute angle and $A = \sin^{-1} \dfrac{4}{\sqrt{41}}$, find the value of tan A.   14. . . . .

15 If tan $x = \frac{1}{2}$ and tan $y = \frac{3}{4}$, find tan $(x + y)$.   15. . . . .

16 If cos $x = \frac{81}{80}$ and x is an acute angle, find cos $\frac{1}{2}$ x.   16. . . . .

17 Express cos $4x$ + cos $2x$ as the product of two functions.   17. . . . .

18 Find the maximum value of 4 cos $\frac{1}{4}$ x.   18. . . . .

19 If the data $A = 50°$, $b = 12$ and $a = 9$ are used (a) no triangle can be constructed, (b) one triangle can be constructed, (c) two triangles can be constructed. Which is correct a, b or c?   19. . . . .

20 If A is an angle in the second quadrant and if cos $A = -\frac{4}{5}$, find the value of sin A.   20. . . . .

[4]