

The University of the State of New York

323D HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Wednesday, January 26, 1955—9.15 a.m. to 12.15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year.

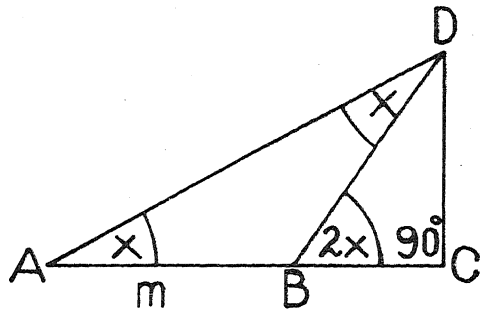
Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from this part. Show all work unless otherwise directed.

- 21 *Derive* the formula for $\sin(x + y)$ where x , y and $(x + y)$ are positive acute angles. [10]
- 22 a Prove the identity: $(1 - \cos A)(1 + \sec A) = \sin A \tan A$. [5]
b Find to the *nearest degree* all the values of x greater than 0° but less than 360° which satisfy the equation:
$$5 \cos^2 x + 9 \cos x - 2 = 0$$
 [5]
- 23 a *Sketch* the graph of $y = \cos \frac{1}{2}x$ as x varies from 0 to 2π radians. [4]
b On the set of axes used in a, *sketch* the graph of $y = \frac{1}{2} \cos x$ as x varies from 0 to 2π radians. [4]
c From the graphs made in answer to a and b determine the number of values of x that satisfy the equation $\frac{1}{2} \cos x = \cos \frac{1}{2}x$ when:
(1) x is between 0 and π radians [1]
(2) x is between π and 2π radians [1]

24 In the figure at the right, DC is perpendicular to AC , angles DAB and ADB are each represented by x , angle DBC is represented by $2x$ and the length of AB by m . The length of each line segment in column I is represented by *one and only one* of the expressions in column II. On your answer paper write the numbers (1)—(5), representing the items in column I, and after *each* number write the *letter* that indicates the corresponding correct expression in column II. [10]



Column I

- (1) BD
- (2) CD
- (3) BC
- (4) AD
- (5) AC

Column II

- (a) m
- (b) $2m \sin x$
- (c) $2m \cos x$
- (d) $m \sin 2x$
- (e) $m \cos 2x$
- (f) $m \cos^2 x$
- (g) $2m \sin^2 x$
- (h) $2m \cos^2 x$

Part III

Answer at least two questions from this part. Show all work.

25 In triangle ABC , angle $A = 37^\circ 20'$, angle $B = 56^\circ 30'$ and side $c = 6760$ feet. Find to the nearest ten feet the length of the altitude to base AB . [10]

26 Point B is directly east of point A . Point C bears $N 67^\circ 20' E$ from A and $N 52^\circ 10' E$ from B . If the distance AC is 957 yards, find to the nearest yard the distance from A to B . [5, 5]

27 Forces of 684 pounds and 976 pounds act upon a body at an angle of $106^\circ 20'$. Find to the nearest ten minutes the angle which the resultant makes with the smaller force. [3, 7]

28 In triangle ABC , $a = 797$, $b = 874$ and $c = 969$. Find angle C to the nearest ten minutes. [10]

Be sure you have answered a total of 5 questions from parts II and III.

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Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

- 1 If $2 \cos^2 A = 1$, find the smallest positive value of A . 1.....
- 2 Express $\tan (-116^\circ)$ as a function of a positive acute angle. 2.....
- 3 In triangle ABC , angle $C = 90^\circ$ and $\sec A = \frac{4}{3}$. Find $\csc B$. 3.....
- 4 Find the smallest positive value of $\sin^{-1} \frac{\sqrt{3}}{2}$. 4.....
- 5 Express in radians an angle of 75° . [Answer may be left in terms of π .] 5.....
- 6 An angle of x radians at the center of a circle intercepts an arc whose length is $3x$. Find the length of the radius of the circle. 6.....
- 7 Find the number whose logarithm is 3.3333. 7.....
- 8 Find $\log \cot 48^\circ 36'$. 8.....
- 9 Find to the *nearest minute* the positive acute angle whose sine is 0.7038. 9.....
- 10 Two forces of 100 pounds and 23 pounds act upon a body at right angles. Find to the *nearest degree* the angle which the resultant makes with the 100-pound force. 10.....
- 11 The sides of a triangle are 2, 3 and 4. Find the cosine of the largest angle of the triangle. 11.....
- 12 In triangle ABC , $a = 9$, $b = 5$ and $C = 60^\circ$. Find $\tan \frac{1}{2}(A - B)$. [Answer may be left in radical form.] 12.....
- 13 Two sides of a triangle are 3 and 4 and the angle between them is A . Express the area of the triangle in terms of $\sin A$. 13.....

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14 Express $\sin A + \sin B$ in terms of a product of two functions. 14.....

15 Find the minimum value of $3 \cos 2x$. 15.....

Directions (16–20): Indicate the correct completion for *each* statement by writing the letter *a*, *b*, or *c* on the line at the right.

16 As A increases from 270° to 360° , (a) $\sin A$ increases and $\cos A$ decreases (b) $\sin A$ decreases and $\cos A$ increases (c) both $\sin A$ and $\cos A$ increase 16.....

17 If a and b are two sides of a triangle opposite angles A and B respectively, then (a) $\log a - \log b = \log \sin A - \log \sin B$
 (b) $\log \left(\frac{a}{b} \right) = \frac{\log \sin A}{\log \sin B}$
 (c) $a \log \sin B = b \log \sin A$ 17.....

18 In the equation $\sin^2 x - 3 \sin x + 2 = 0$, the number of solutions for x between 0° and 180° is (a) 0 (b) 1 (c) 2 18.....

19 The expression $\frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$ can be reduced to (a) $\cos \frac{1}{2}A$
 (b) $\cos A$ (c) $\cos 2A$ 19.....

20 If $\cos A = x$, then $\sin^2 \frac{1}{2}A$ is (a) $\frac{1-x}{2}$ (b) $\frac{1+x}{2}$
 (c) $\frac{1-x}{1+x}$ 20.....

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING TRIGONOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 16–20, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

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| (1) 45° | (12) $\frac{2\sqrt{3}}{7}$ |
| (2) $\tan 64^\circ$ or $\cot 26^\circ$ | (13) $6 \sin A$ |
| (3) $\frac{4}{3}$ | (14) $2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$ |
| (4) 60° | (15) -3 |
| (5) $\frac{5\pi}{12}$ | (16) <i>c</i> |
| (6) 3 | (17) <i>a</i> |
| (7) 2154 | (18) <i>b</i> |
| (8) 9.9453–10 | (19) <i>c</i> |
| (9) $44^\circ 44'$ | (20) <i>a</i> |
| (10) 13° | |
| (11) $-\frac{1}{4}$ | |