

January 23, 1957

## Part I

Answer all questions in this part. Each correct answer will receive  $2\frac{1}{2}$  credits.

1. Find the number of inches in the radius of a circle in which a central angle of 4 radians intercepts an arc of 6 inches. 1\_\_\_\_\_
  2. What is the value of  $\cot \frac{5}{4} \pi$ ? 2\_\_\_\_\_
  3. Find the positive value of  $\tan \left( \arcsin \frac{3}{5} \right)$ . 3\_\_\_\_\_
  4. Express  $\sin(x - y)$  in terms of the sine and cosine of  $x$  and of  $y$ . 4\_\_\_\_\_
  5. If  $A$  is a positive acute angle and  $\cos A = \frac{7}{9}$ , find the value of  $\sin \frac{1}{2}A$ . 5\_\_\_\_\_
  6. In triangle  $ABC$ ,  $a = 5$ ,  $b = 9$  and  $\sin A = 0.30$ . Find the value of  $\sin B$ . 6\_\_\_\_\_
  7. In triangle  $ABC$ ,  $a = 4$ ,  $b = 5$  and  $c = 6$ . Find the value of  $\cos A$ . 7\_\_\_\_\_
  8. In triangle  $ABC$ ,  $a = 6$ ,  $b = 4$  and  $\tan \frac{1}{2}(A - B) = 0.50$ . Find the value of  $\tan \frac{1}{2}(A + B)$ . 8\_\_\_\_\_
  9. What is the area of triangle  $ABC$  if  $a = 5$ ,  $b = 6$ ,  $\sin C = 0.6$ ,  $\cos C = 0.8$  and  $\tan C = 0.75$ ? 9\_\_\_\_\_
  10. Solve the equation  $\sin^2 \frac{1}{2}x = 1$  for the smallest positive value of  $x$ . 10\_\_\_\_\_
  11. If  $\log n = 2.3571$ , find  $n$ . 11\_\_\_\_\_
  12. Find  $\log \sin 61^\circ 43'$ . 12\_\_\_\_\_
  13. Find to the nearest minute the positive acute angle whose cosine is 0.9030. 13\_\_\_\_\_
  14. From an observation post 108 feet directly above a point  $A$  on level ground, the angle of depression of point  $B$  on the ground is observed to be  $6^\circ 10'$ . Find the distance from  $A$  to  $B$  to the nearest foot. 14\_\_\_\_\_
- Directions (15-20): Indicate the correct completion for each of the following by writing on the line at the right the letter  $a$ ,  $b$ ,  $c$  or  $d$ .
15. If  $\sin \theta$  is negative and  $\tan \theta$  is positive, then  $\theta$  is an angle in the (a) first quadrant (b) second quadrant (c) third quadrant (d) fourth quadrant 15\_\_\_\_\_
  16. The function  $\csc 270^\circ$  (a) has a value of  $-1$  (b) has a value of 0 (c) has a value of 1 (d) is not defined 16\_\_\_\_\_

17. The function  $\cos 250^\circ$  is equal to (a)  $\sin 20^\circ$   
 (b)  $-\sin 20^\circ$  (c)  $\sin 70^\circ$  (d)  $-\sin 70^\circ$  17\_\_\_\_\_
18. If  $m > 1$ , the maximum value of  $2m \sin 2x$  is  
 (a) 2 (b)  $m$  (c)  $2m$  (d)  $4m$  18\_\_\_\_\_
19. Using the data  $A = 35^\circ$ ,  $b = 3$  and  $a = 4$ , it is possible to construct (a) two triangles (b) a right triangle (c) no triangle (d) an obtuse triangle 19\_\_\_\_\_
20. As  $x$  varies from  $45^\circ$  to  $315^\circ$ , the graph of  $y = \sin 2x$   
 (a) does not cross the  $x$ -axis (b) crosses the  $x$ -axis once  
 (c) crosses the  $x$ -axis twice (d) crosses the  $x$ -axis three times 20\_\_\_\_\_

## Part II

*Answer three questions from this part. Show all work unless otherwise directed.*

21. *a* Starting with the formula for  $\cos(x + y)$ , derive the formula for  $\cos 2x$  in terms of  $\cos x$ . [4]

*b* Starting with the formula for  $\cos 2x$  in terms of  $\cos x$ , derive the formula for  $\cos \frac{1}{2}A$  in terms of  $\cos A$ . [6]

22. *a* Prove that the following equation is an identity: [5]

$$\frac{2 \cot x}{1 + \cot^2 x} = \sin 2x$$

- b* Show that  $\frac{\sin 7x + \sin 5x}{\cos 7x - \cos 5x}$  may be reduced to  $-\cot x$ . [5]

23. Find to the nearest degree all values of  $x$  between  $0^\circ$  and  $360^\circ$ , inclusive, that satisfy the equation  $3 \cos^2 x + 4 \sin x + 1 = 0$ . [10]

24. *a* Draw the graph of  $y = \tan x$  as  $x$  varies from  $-90^\circ$  to  $+90^\circ$ , assigning to  $x$  the values  $0^\circ, \pm 30^\circ, \pm 45^\circ, \pm 60^\circ, \pm 90^\circ$ . [4]

*b* On the same set of axes used in part *a*, draw the graph of  $y = \cos x$ , using the same interval and the same set of values for  $x$ . [4]

*c* From the graphs made in answer to parts *a* and *b*, determine the number of values of  $x$  that satisfy the equation  $\tan x = \cos x$  when

- (1)  $x$  is between  $0^\circ$  and  $45^\circ$  [1]  
 (2)  $x$  is between  $45^\circ$  and  $90^\circ$  [1]

25. The rectangular coordinates of the point  $P$  are represented by  $x$  and  $y$ . The distance from the origin  $O$  to  $P$  is represented by  $r$ , and the angle that  $OP$  makes with the positive portion of the  $x$ -axis is represented by  $\theta$ .

*a* Express  $x$  in terms of  $r$  and a trigonometric function of  $\theta$ . [2]

*b* Express  $y$  in terms of  $r$  and a trigonometric function of  $\theta$ . [2]

*c* Using the results obtained in parts *a* and *b*, show that the equation  $x^2 - y^2 = 4$  can be reduced to the form  $r^2 \cos 2\theta = 4$ . [6]

## Part III

*Answer two questions from this part. Show all work.*

26. In triangle  $ABC$ ,  $a = 959$ ,  $b = 631$  and  $C = 68^\circ$ . Find  $A$  to the nearest degree. [10]

27. Two forces of 437 pounds and 876 pounds, respectively, act upon a body at an acute angle with each other. The angle between the resultant force and the 437-pound force is  $41^\circ 10'$ . Find to the nearest ten minutes the angle formed by the 437-pound and the 876-pound force. [4, 6]

28. Airport  $A$  is 250 miles directly west of airport  $C$ , and airport  $B$  is 101 miles directly north of airport  $C$ . An airplane flies from  $C$  in the direction  $N 35^\circ W$  to a point  $D$ , which is on the direct path from  $A$  to  $B$ . Find the distance from  $C$  to  $D$  to the nearest mile. [4, 6]

29. The sides of a triangle are 5.46, 6.87 and 7.65. Find the smallest angle of the triangle to the nearest degree. [10]