The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# TRIGONOMETRY

Thursday, January 25, 1962 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

## Part I

**Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of $\pi$ or in radical form.**

1 Find the numerical value of $\cos\left(-\frac{\pi}{3}\right)$. 1.....

2 The radius of a circle is 9 inches. Find the number of radians in a central angle which subtends an arc of 1 foot in this circle. 2.....

3 Find the value of $\sec\left(\arccos\frac{12}{13}\right)$. 3.....

4 Find the antilogarithm of 2.8415. 4.....

5 Find log cos 63° 34′. 5.....

6 If $\sin\theta = t$, express $\cos(270° + \theta)$ in terms of $t$. 6.....

7 If $x$ is a positive acute angle, express csc $x$ in terms of tan $x$. 7.....

8 Find the number of degrees in $\theta$ if $\sin\theta = -\frac{\sqrt{2}}{2}$ and tan $\theta$ is negative. 8.....

9 The resultant of two forces acting at right angles (one horizontal and one vertical) is a force of 200 pounds which makes an angle of 22° with the vertical force. Find to the *nearest pound* the value of the horizontal force. 9.....

10 If $\tan\theta = \frac{3}{4}$ and $\theta$ terminates in the third quadrant, find the value of sec $\theta$. 10.....

11 In triangle $ABC$, $a = 3$, $b = 7$ and $c = 9$. Find the cosine of the smallest angle in the triangle. 11.....

12 In triangle $ABC$, $A = 30°$, $B = 135°$ and $a = 10$. Find $b$. 12.....

13 Express tan $2x$ in terms of tan $x$. 13.....

14 Express in degrees the smallest positive value of $x$ that satisfies the equation $\tan^2 3x - 1 = 0$. 14.....



[OVER]

15 In triangle $ABC$, $a = 11$, $b = 9$ and $C = 40°$. Find to the *nearest hundredth* the value of $\tan \frac{1}{2}(A - B)$. 15.....

*Directions* (16–25): Indicate the correct completion for *each* of the following by writing on the line at the right the number 1, 2, 3 or 4.

16 If $A$ is an acute angle, $\log \sec A$ equals

(1) $\frac{1}{\log \cos A}$ (3) $\log \csc A$

(2) $-\log \cos A$ (4) $\log \cos A$ 16.....

17 An angle of $\frac{2}{3}\pi$ radians has the same terminal side as the angle whose measure is

(1) $\frac{1}{3}\pi$ radians (3) $\frac{4}{3}\pi$ radians

(2) $-\frac{1}{3}\pi$ radians (4) $-\frac{4}{3}\pi$ radians 17.....

18 The maximum value of $\frac{1}{2}\cos 2x$ is

(1) 1 (3) $\frac{1}{2}$

(2) 2 (4) 4 18.....

19 The expression $\cos \frac{3}{2}x + \cos \frac{1}{2}x$ is equal to

(1) $2 \sin x \cos \frac{1}{2}x$ (3) $2 \cos x \cos \frac{1}{2}x$

(2) $\cos 2x$ (4) $2 \cos 2x \cos x$ 19.....

20 If the data $B = 36°$, $b = 6$ and $c = 10$ are used, triangle $ABC$

(1) must be a right triangle
(2) must be an acute triangle
(3) must be an obtuse triangle
(4) may be either an acute triangle or an obtuse triangle 20.....

21 As $x$ increases from 180° to 360°, $\cos x$

(1) increases (3) increases, then decreases
(2) decreases (4) decreases, then increases 21.....

22 If $\theta$ is a positive obtuse angle and $\cos\theta = -\frac{3}{5}$, the value of $\tan\frac{1}{2}\theta$ is

(1) $-2$
(2) $2$
(3) $-\frac{1}{2}$
(4) $\frac{1}{2}$

22.....

23 If $\sin x = \cos y = a$ and $\cos x = \sin y = b$, then $\cos(x - y)$ equals

(1) 0
(2) $a^2 - b^2$
(3) $b^2 - a^2$
(4) $2ab$

23.....

24 Tan $x$ is *not* defined when

(1) $\cos x = 0$
(2) $\sin x = 0$
(3) $\cot x = 1$
(4) $x = 0$

24.....

25 The expression $\frac{\sin 2x}{\sin(-x)}$ is equivalent to

(1) $2\sin x$
(2) $-2\sin x$
(3) $2\cos x$
(4) $-2\cos x$

25.....

26 Find the area of a triangle whose sides are 5, 6 and 7. 26.....

27 For what value of $k$ is the period of the function $y = 3\sin kx$ equal to 120°? 27.....

*Directions* (28–30): Indicate whether the following statements are true for

(1) all real values of $x$,
(2) some, but not all, real values of $x$,
(3) no real value of $x$,

by writing on the line at the right the *number* 1, 2 or 3.

28 $\cos^2 x > 1 + \sin^2 x$ 28.....

29 $\cos 2x = \sin\left(\frac{\pi}{2} - 2x\right)$ 29.....

30 $\frac{1}{2}\sin^2 2x + \frac{1}{2}\cos^2 2x = 1$ 30.....



[OVER]

**Part II**

**Answer four questions from this part. Show all work unless otherwise directed.**

31 Find all the values of $\theta$ between $0°$ and $360°$ that satisfy the equation $2\cos 2\theta - 7\cos\theta = 0$. [Express the values of $\theta$ to the *nearest degree.*] [10]

32 *a* Starting with a formula for $\cos 2A$, *derive* a formula for $\sin \frac{1}{2}x$ in terms of $\cos x$. [6]

*b* Show that the expression $\tan y\,(\csc y - \sin y)$ is equivalent to $\cos y$. [4]

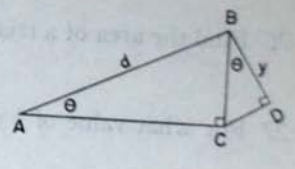
33 *a* *Sketch* the graph of $y = 2\cos \frac{1}{2}x$ as $x$ varies from $-\pi$ radians to $+\pi$ radians. [5]

*b* On the same set of axes, *sketch* the graph of $y = \sin x$ as $x$ varies from $-\pi$ radians to $+\pi$ radians. [3]

*c* On the graph, draw and label a line segment $AB$ whose length is $2\cos \frac{1}{2}x - \sin x$ when $x = \frac{\pi}{2}$. [2]

34 The captain of a ship sights a lighthouse on a bearing 060° (N 60° E) and at the same time sights a buoy on a bearing 051° (N 51° E). He knows the buoy is 1.3 miles from the lighthouse on a bearing of 340° (N 20° W) from the lighthouse to the buoy. Find to the *nearest tenth of a mile* the distance from the ship to the buoy. [6, 4]

35 In the accompanying diagram, $BC \perp AC$, $BD \perp CD$ and $\angle BAC = \angle CBD = \theta$. Using the letters as shown on the diagram, *derive* the relationship $y = \frac{1}{2}d \sin 2\theta$. [10]




36 Answer *either* *a* or *b*: [10]

*a* In triangle $ABC$, $a = 15.6$, $b = 23.7$ and $c = 10.3$. Find to the *nearest degree* the largest angle of the triangle.

*OR*

*b* Two forces of 259 pounds and 615 pounds, respectively, act on a body at an angle of 44° 40′ with each other. Find to the *nearest ten minutes* the angle formed by the resultant and the smaller force.

# FOR TEACHERS ONLY

## INSTRUCTIONS FOR RATING

## TRIGONOMETRY

**Thursday**, January 25, 1962 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 16–25, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{1}{2}$

(2) $\frac{4}{3}$

(3) $\frac{13}{12}$

(4) 694.2

(5) 9.6485 — 10

(6) $t$

(7) $\frac{\sqrt{1 + \tan^2 x}}{\tan x}$

(8) 315

(9) 75

(10) $-\frac{5}{4}$

(11) $\frac{121}{126}$

(12) $10\sqrt{2}$ *or* 14.1

(13) $\frac{2 \tan x}{1 - \tan^2 x}$

(14) 15

(15) 0.27

(16) 2

(17) 4

(18) 3

(19) 3

(20) 4

(21) 1

(22) 2

(23) 4

(24) 1

(25) 4

(26) $6\sqrt{6}$ *or* 14.7

(27) 3

(28) 3

(29) 1

(30) 3

[OVER]

**Part II**

*Please refer to the Department's pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 104°, 256° [10]

(34) Analysis [6]
8.2 [4]

(36) *a* 131° [10]
*b* 31° 50′ [10]