June 20, 1956

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Express sec 290° as a function of a positive acute angle. 1_____

2. Express in radians an angle of 100°. 2_____

3. Find the smallest positive value of A which satisfies the equation $\tan^2 A - 3 = 0$. 3_____

4. In triangle ABC, angle $C = 90°$, side $a = 18$ and side $b = 12$. Find $\tan \frac{1}{2}(A - B)$. 4_____

5. Find the positive value of $\cos \left\{ \sin^{-1} \frac{5}{13} \right\}$ 5_____

6. If $\tan \theta = \frac{1}{2}$, find $\sin 2\theta$. 6_____

7. Express $\sin^2 \frac{\theta}{2}$ in terms of $\cos \theta$. 7_____

8. Find the antilogarithm of 1.8362 8_____

9. Find log cot 53° 27′. 9_____

10. The legs of a right triangle are 12 and 24. Find to the *nearest minute* the smaller acute angle of the triangle. 10_____

Directions (11-16): Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* on the line at the right.

11. The expression $\frac{\tan \theta + \sec \theta}{1 + \sin \theta}$ can be reduced to

(a) $\cos \theta$ (b) $\frac{1}{\cos \theta}$ (c) $\sin \theta$ 11_____

12. The expression $\sin (45° + x)$ is always equal to

(a) $\frac{1}{\sqrt{2}} + \sin x$ (b) $\frac{\sin x - \cos x}{\sqrt{2}}$ (c) $\frac{\cos x + \sin x}{\sqrt{2}}$ 12_____

13. Using the data $A = 43°$, $a = 15$ and $b = 24$, it is possible to construct (a) no triangle (b) only one triangle (c) two triangles 13_____

14. The expression $\sin^2 x - \cos^2 x$ is always equal to
(a) $\cos(-2x)$ (b) -1 (c) $-\cos 2x$ 14_____

15. If, in triangle ABC, $a = 3$, $b = 5$ and $c = 6$, the cosine of the largest angle is equal to (a) $-1/15$ (b) $-5/6$ (c) $1/9$ 15_____

16. The graph of the function $y = 2 \sin \frac{1}{2} x$ passes through the point whose coordinates are (a) $(\pi, 2)$ (b) $(\frac{\pi}{2}, 1)$
(c) $(\pi, 1)$ 16_____

Directions (17-20): For *each* of the following, tell whether the statement is *always* true, *sometimes* true or *never* true by writing the word *always*, *sometimes* or *never* on the line at the right.

17. $\cos(270° - \theta) = -\sin\theta$. 17_____

18. $\sin^2 2x + \cos^2 2x = 2$. 18_____

19. $\cos 4A - \cos 2A = -2 \sin 3A \sin A$. 19_____

20. $\cot^2 A - \cot A = 0$. 20_____

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21. *a* Prove the identity: $\dfrac{\cot^2\theta}{\csc\theta - 1} = \dfrac{1 + \sin\theta}{\sin\theta}$. [6]

b Find the smallest positive value of A which satisfies the equation $2\sin^2 A + \sin A - 1 = 0$. [4]

22. Answer both *a* and *b* without the use of trigonometric tables.

a Angles x and y are acute, $\sin x = 4/5$ and $\cos y = 5/13$. Find the value of $\tan(x + y)$. [5]

b If $A = \tan^{-1}\dfrac{2}{\sqrt{5}}$, find the value of $\sin 2A$.

[Answer may be left in radical form.] [5]

23. Derive the law of cosines. [Consider only the case in which the triangle is acute.] [10]

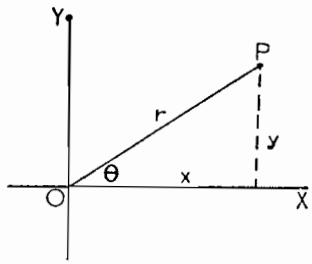
24. *a* On the same set of axes *sketch* the graph of $y = \sin x$ and $y = \cos x$ as x varies from 0 to 2π radians. [3, 3]

b From the graphs made in answer to part *a*, determine the *smallest* value of x *greater than* 0 *and less than* 2π *radians* for which

(1) $\sin x + \cos x = 1$ [2]

(2) $\sin x + \cos x = -1$ [2]

25. P is a point whose rectangular coordinates are represented by x and y as shown in the drawing. The distance from the origin O to point P is represented by r, and the angle that OP makes with the positive portion of the x-axis is represented by θ.



a Express x in terms of r and θ. [1]

b Express y in terms of r and θ. [1]

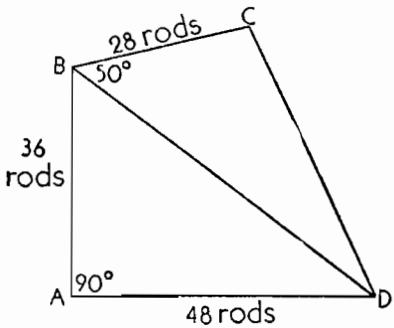
c Show that the equation $x^2 + y^2 - 2x = 0$ can be reduced to the form $r = 2\cos\theta$. [8]

Answer two questions from this part. Show all work.

26. In triangle ABC, angle $C = 78°$, side $AC = 150$ feet and side $BC = 200$ feet. Find angle A to the *nearest degree.* [10]

27. In triangle ABC, angle $C = 90°$, angle $CAB = 48°$ and side $AB = 285$. P is a point on side CB and angle $CAP = 22°$. Find PB to the *nearest integer.* [10]

28. A plot of ground has the form of the quadrilateral $ABCD$ shown at the right. Side $AB = 36$ rods, side $AD = 48$ rods, angle $DAB = 90°$, angle $DBC = 50°$ and side $BC = 28$ rods. Find to the *nearest square rod* the area of the plot. [10]



29. Point B is 12 miles directly east of point A. Point C, which is north and east of A, is 18 miles from A and 14 miles from B. Find to the *nearest degree* the bearing of C from A. [10]