

TRIGONOMETRY

Wednesday, June 22, 1960 — 9:15 a.m. to 12:15 p.m., only

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of  $\pi$  or in radical form.

- 1 Find the numerical value of  $\cos \frac{\pi}{3}$ . 1.....
- 2 Find the number of inches in the radius of a circle in which a central angle of  $1\frac{1}{2}$  radians subtends an arc of 6 inches. 2.....
- 3 Find in degrees the smallest positive value of  $A$  which satisfies the equation  $4 \sin^2 A - 1 = 0$ . 3.....
- 4 If  $7 \tan A - 3 = 0$ , express  $A$  in inverse trigonometric form. 4.....
- 5 Find the antilogarithm of 1.3799. 5.....
- 6 Find  $\log \tan 36^\circ 28'$ . 6.....
- 7 Find  $\cos 75^\circ 34'$ . 7.....
- 8  $A$  is 100 miles N  $42^\circ$  E of  $B$ .  $C$  is due north of  $B$  and due west of  $A$ . Find to the nearest mile the distance from  $B$  to  $C$ . 8.....
- 9 In triangle  $ABC$ ,  $a = 8$ ,  $c = 5$  and  $B = 20^\circ$ . Find to the nearest integer the area of triangle  $ABC$ . 9.....
- 10 In triangle  $ABC$ ,  $b = 12$ ,  $c = 6$  and  $A = 100^\circ$ . Find to the nearest hundredth the value of  $\tan \frac{1}{2}(B - C)$ . 10.....
- 11 In triangle  $ABC$ ,  $a = 6$ ,  $b = 7$  and  $\cos C = \frac{1}{4}$ . Find  $c$ . 11.....
- 12 In triangle  $ABC$ ,  $a = 15$ ,  $b = 6$  and  $A = 30^\circ$ . Find  $\sin B$ . 12.....
- 13 Express  $\cos(x - y)$  in terms of the sine and cosine of  $x$  and  $y$ . 13.....
- 14 If  $A$  is a positive acute angle, express  $\cos A$  in terms of  $\cot A$ . 14.....
- 15 If  $x$  is an acute angle and  $\cos x = m$ , express  $\cos \frac{x}{2}$  in terms of  $m$ . 15.....

- 16 Express  $160^\circ$  in radian measure. 16.....
- 17 Find in degrees the acute angle  $x$  if  $\sin x = \cos (3x - 10^\circ)$ . 17.....
- 18 Express  $\cos \theta \cot \theta$  in terms of  $\sin \theta$ . 18.....
- 19 Express  $\log \tan A$  in terms of  $\log \sin A$  and  $\log \cos A$ . 19.....
- 20 Given  $\sec 50^\circ = a$ , express  $130^\circ$  in terms of  $a$ . 20.....

*Directions (21–28):* Indicate the correct completion for each of the following by writing on the line at the right the number 1, 2, 3 or 4.

- 21 The expression  $\tan (45^\circ + x)$  is equal to (1)  $\frac{1 + \tan x}{1 - \tan x}$   
 (2)  $\frac{1 - \tan x}{1 + \tan x}$  (3)  $1 + \tan x$  (4)  $1 - \tan x$  21.....
- 22 The minimum value of  $2 \cos 3x$  is (1)  $-1$  (2)  $2$  (3)  $-6$   
 (4)  $-2$  22.....
- 23 The expression  $\cos 3x + \cos x$  is equal to (1)  $\cos 4x$   
 (2)  $2 \cos 2x \sin x$  (3)  $2 \cos 2x \cos x$  (4)  $-2 \sin 2x \sin x$  23.....
- 24 For all values of  $x$ ,  $\cos (-x) + \sin (-x)$  is equal to  
 (1)  $\cos x + \sin x$  (2)  $-\cos x + \sin x$  (3)  $-\cos x - \sin x$   
 (4)  $\cos x - \sin x$  24.....
- 25 If both  $\sin x$  and  $\cos x$  increase as  $x$  increases, then  $x$  must be an angle in quadrant (1)one (2)two (3)three (4)four 25.....
- 26 The graph of the function  $y = 2 \cos \frac{1}{2}x$  passes through the point whose coordinates are (1)  $(\pi, 2)$  (2)  $(2\pi, 2)$  (3)  $(\pi, -2)$   
 (4)  $(2\pi, -2)$  26.....
- 27 Using the data  $A = 40^\circ$ ,  $a = 13$  and  $b = 20$ , (1)triangle  $ABC$   
 must be acute (2)triangle  $ABC$  must be obtuse (3)triangle  $ABC$   
 may be either acute or obtuse (4)no triangle can be constructed 27.....
- 28 For all values of  $x$ ,  $\sin (270^\circ + x)$  is equal to (1)  $-\sin x$   
 (2)  $-\cos x$  (3)  $\cos x$  (4)  $-\csc x$  28.....

*Directions (29–30):* Indicate the correct completion for each of the following by writing on the line at the right the number 1, 2 or 3.

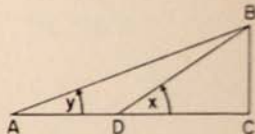
- 29 The equation  $\cos 2x + 1 = 2 \cos^2 x$  is true for (1)all values of  $x$   
 (2)some but not all values of  $x$  (3)no value of  $x$  29.....
- 30 The statement  $\sin 2x > 2 \sin x$  is true for (1)all values of  $x$  in  
 quadrant I (2)some but not all values of  $x$  in quadrant I  
 (3)no value of  $x$  in quadrant I 30.....

## Part II

Answer four questions from this part. Show all work unless otherwise directed.

- 31 Find all values of  $A$  between  $0^\circ$  and  $360^\circ$  that satisfy the equation  $2 \cos 2A - 3 \sin A - 1 = 0$ .  
[Express approximate values of  $A$  to the nearest degree.] [10]
- 32 a Starting with the formulas for  $\sin(x - y)$  and  $\cos(x - y)$ , derive the formula for  $\tan(x - y)$ . [6]
- b Show that the expression  $\frac{\sec x}{\cot x + \tan x}$  can be reduced to  $\sin x$ . [4]
- 33 a On the same set of axes, sketch the graphs of  $y = \sin \frac{1}{2}x$  and  $y = 2 \cos x$  as  $x$  varies from  $0$  to  $2\pi$  radians. [Label each curve with its equation.] [4, 4]
- b From the graphs made in answer to a, find the number of values of  $x$  greater than  $0$  and less than  $2\pi$ , for which  $2 \cos x - \sin \frac{1}{2}x = 0$ . [2]

- 34 Given right triangle  $ABC$ , hypotenuse  $AB$ ,  $D$  any point on  $AC$  and line  $BD$  drawn. Derive a formula for  $BC$  in terms of  $AD$ , angle  $x$  and angle  $y$ . [10]



- 35 The captain of a ship sights a lighthouse bearing  $040^\circ$  (N  $40^\circ$  E). After sailing on a course  $335^\circ$  (N  $25^\circ$  W) for a distance of 5.5 miles, he then finds the bearing of the lighthouse is  $075^\circ$  (N  $75^\circ$  E). Find to the nearest tenth of a mile the distance of the ship from the lighthouse at the time the second bearing was taken. [6, 4]
- 36 Answer either a or b:
- a In triangle  $ABC$ ,  $a = 19.5$ ,  $b = 28.7$  and  $c = 17.6$ . Find to the nearest degree the smallest angle of triangle  $ABC$ . [10] OR
- b Two forces of 70 pounds and 125 pounds act on a body at an angle of  $68^\circ$  with each other. Find to the nearest ten minutes the angle formed by the lines of action of the resultant and the larger force. [10]

# FOR TEACHERS ONLY

## INSTRUCTIONS FOR RATING

### TRIGONOMETRY

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Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

#### Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 5 is not expressed to four significant digits. For questions 21–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1)  $\frac{1}{2}$

(2) 4

(3) 30

(4)  $\arcsin \frac{1}{2}$  or  $\sin^{-1} \frac{1}{2}$

(5) 23.98

(6) 9.8687—10 or 0.1313

(7) 0.2493

(8) 74

(9) 7

(10) 0.28

(11) 8

(12)  $\frac{1}{2}$

(13)  $\cos x \cos y + \sin x \sin y$

(14)  $\frac{\cot A}{\sqrt{1 + \cot^2 A}}$

(15)  $\sqrt{\frac{1+m}{2}}$

(16)  $\frac{8\pi}{9}$  or 2.8

(17) 25

(18)  $\frac{1 - \sin^2 \theta}{\sin \theta}$

(19)  $\log \sin A - \log \cos A$

(20)  $-\frac{1}{a}$

(21) 1

(22) 4

(23) 3

(24) 4

(25) 4

(26) 4

(27) 3

(28) 2

(29) 1

(30) 3

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

## Part II

(31)  $14^\circ, 166^\circ, 270^\circ$  [10]

(33)  $b$  two [2]

(34)  $\frac{AD \sin x \sin y}{\sin(x-y)}$  or  $\frac{AD \tan x \tan y}{\tan x - \tan y}$  or  $\frac{AD}{\cot y - \cot x}$

(35) Analysis [6]  
8.7 [4]

(36)  $a$   $37^\circ$  [10]

OR

$b$   $23^\circ 20'$  [10]