TRIGONOMETRY

Thursday, August 24, 1950 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in trigonometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1950 or number and length in minutes of lessons taken in the summer of 1950 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1950 or an equivalent program of tutoring approved in advance by the Department is required.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 a Prove the identity: $\dfrac{\sin 2A}{\sin A} = \dfrac{\cos 2A + 1}{\cos A}$ [5]

b Solve for the positive value of x less than $360°$: $1 + 2 \csc x = \sin x$ [5]

22 a On the same set of axes, sketch the graphs of $y = \sin x$ and $y = 2 \cos x$ as x varies from 0 to 2π radians.   [3, 5]

b From the graphs made in answer to a, determine the quadrants in which can be found the values of x satisfying the equation: $\sin x = 2 \cos x$ [2]

23 Derive the law of cosines for the case in which the triangle is acute.   [10]

24 In the figure at the right, $\angle C = 90°$, $\angle DAC$ is x and $\angle BAD$ is $2x$. Show that $DB = 2\,AB \sin x$.   [10]
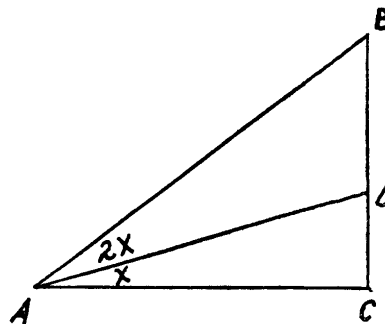
[1]   [OVER]

Part III

Answer at least two questions from part III.

25 In $\triangle ABC$, $a = 28.4$, $b = 32.5$, $c = 36.3$. Find C to the *nearest minute*.   [10]

26 In order to find the distance across a river, a surveyor uses points A and B along the bank of the river and a point C on the opposite bank. He finds angle CAB to be 62° 10′, angle ABC to be 40° 30′ and AB to be 275 feet. Find, to the *nearest foot*, the width of the river.   [3, 7]

27 Starting from a position A, a ship sails a certain distance in the direction S 70° 20′ E from A, until it reaches a position B. It then takes the direction N 37° 10′ E from B and sails 194 miles to its destination C. If C is N 65° 50′ E of A, find AC to the *nearest mile*.   [6, 4]

28 In $\triangle ABC$, $A = 57° 40′$, $b = 93.7$ and $c = 72.3$. Find B to the *nearest minute*.   [10]

Fill in the following lines:

Name of pupil..Name of school ....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Find the logarithm of 3.064   1................

2 Find the number whose logarithm is 9.8914 —10   2................

3 Find log tan 72° 13′   3................

4 If cos A = 0.7946, find acute angle A to the *nearest minute*.   4................

5 Express sec (—130°) as a function of a positive acute angle.   5................

6 Express 40° to the *nearest tenth* of a radian.   6................

7 Express as a common fraction the positive value of cos $\left(\sin^{-1} \frac{\sqrt{5}}{3} \right)$ 7................

8 If A is a positive acute angle, express cot A in terms of sin A.   8................

9 In $\triangle ABC$, $a = 2$, $b = 4$, $c = 3$. Find $\cos B$.   9................

10 In $\triangle ABC$, $A = 45°$, and $B = 105°$. Find the numerical

value of $\frac{a}{c}$. [Answer may be left in radical form.]   10................

11 In $\triangle ABC$, $a = 3$, $b = 2$, and tan $\frac{A-B}{2} = .2$. Find the value

of tan $\frac{A+B}{2}$.   11................

12 If $\cos A = m$, express $\sin^2 \frac{A}{2}$ in terms of m.   12................

13 Express tan $(A + B)$ in terms of tan A and tan B.   13................

14 If $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$, and x and y are first quadrant

angles, find cos $(x + y)$.   14................

15 Sides 5 and 12 of a parallelogram include an angle of 150°. Find the area of the parallelogram.   15................

16 A ship, now at a certain position, must sail 120 miles in a direction S 31° E in order to make port. Find, to the *nearest mile*, how far west of its port the ship is now.   16................

17 Express sin 70° — sin 10° as a function of 40°.   17................

Directions (questions 18–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter a, b or c.

18 Using the data $A = 28°$, $a = 12$, $b = 18$, it is possible to construct   (a) only one triangle   (b) two different triangles   (c) no triangle   18......

19 As angle x increases from $\frac{\pi}{2}$ to π radians, cos x (a) increases from —1 to 0

(b) decreases from 1 to 0   (c) decreases from 0 to —1   19......

20 When drawn on the same set of axes, the graph of $y = 2$ *will never* intersect the graph of   (a) $y = 3 \sin x$,   (b) $y = \sin 3x$ (c) $y = \tan x$ 20......

[3]